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The Logic of God

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Introduction[1]

Let us begin with the following quotation from Avicenna's *Metaphysics*:

> As for the obstinate, he must be plunged into fire, since fire and non-fire are identical. Let him be beaten, since suffering and not suffering are the same. Let him be deprived of food and drink, since eating and drinking are identical to abstaining.[2]

This quotation testifies to the fact that the principle of non-contradiction is so firmly implanted in our thinking that we are inclined to treat it as a fundamental ontological requirement reflecting the deepest property, almost a synonym, of reality. And indeed, Avicenna's persuasive argument is, in fact, but a practical implementation of the Aristotle's conviction that some principles, like the principle of non-contradiction, cannot be proved but only argued for per reduction *ad absurdum*.[3] Aristotle treated this principle not only as a law of our reasoning, but also as an ontological law. He developed an extensive analysis of this principle

1 This paper was presented during the 2nd World Congress on Logic and Religion on 22 June 2017 at the University of Warsaw. The Polish version of this paper had been included in a book: M. Heller, *Ważniejsze niż wszechświat*, Copernicus Center Press, Kraków 2018.

2 *Metaphysics*, 1,8, 53.13e–15; quoted after L.R. Horn, *Contradiction*, in: *The Stanford Encyclopedia of Philosophy*, ed. E.N. Zalta, https://plato.stanford.edu/entries/contradiction/.

3 Aristotle, *Metaphysics*, IV, 106a.

in the context of his dispute with Heraclitus who claimed that in the very concept of motion contradiction is involved. Aristotle, on the contrary, argued that motion, being real must be, and in fact is, free of any contradiction. Negation of the non-contradiction principle would also produce ontological, albeit nonsensical, results – the sort of an "ontological overfill of the world", as put by Aristotle:

> Again, if all contradictory statements are true of the same subject at the same time, evidently all things will be one. For the same thing will be a trireme, a wall, and a man, if of everything it is possible either to affirm or to deny anything.[4]

Just as from a proposition and its negation any proposition follows, a contradiction implemented in a world would result in "anything goes". This is why, according to Aristotle, the non-contradiction principle is an ontological statement, tacitly assumed by any science about reality.[5]

In Leibniz's view, the non-contradiction principle is an "*a priori* truth". By an "*a priori* truth" Leibniz understands a "truth" that can be reduced to an identity. To prove that a proposition is an *a priori* truth is to show that its predicate is (implicitly) contained in its subject. There are necessary and contingent truths. A truth is contingent if it cannot be reduced to identity in a finite sequence of steps. However this distinction is only *quoad* us. In fact, all truths are *a priori* since for God an infinite number of steps is no obstacle. As Leibniz explains,

> In the case of contingent truths, even though the predicate is really in the subject, yet one never arrives at a demonstration or an identity, even though the resolution of each term is continued indefinitely. In such cases it is only God, who comprehends the infinite at once, who can see how the one is in the other.[6]

Leibniz himself, and all thinkers who followed his tradition (and in fact almost all Medieval philosophers and theologians before him) had no doubts that God in His deductions (even if they are instantaneous) employs classical logic – the best logic they had at their disposal, and practically the only one they knew.

[4] *Metaphysics*, IV, 107b–108a (translated by W. D. Ross).
[5] See R. Poczobut, *Spór o zasadę sprzeczności*, Towarzystwo Naukowe KUL, Lublin 2000, p. 27.
[6] "Necessary and Contingent Truths", in: G.W. Leibniz, *Philosophical Writings*, ed. G.H.R. Parkinson, Everman's Library, Dent and Sons, London 1973, p. 97.

Moreover, logic itself seems to have something of the absolute. For the logicians of the Leibniz tradition it is natural to somehow identify logic with God as, say, his way of thinking. This absoluteness of logic is especially transparently seen, if we apply Leibniz's famous question "Why is there something rather than nothing?" to logical laws, for instance, to the principle of non-contradiction.

> It can seem clear enough that our cosmos couldn't have existed unless it had first been real that no absurdity was involved. Not first through being earlier in time, but first as a prerequisite, and as prerequisite which wouldn't have depended on the actual existence of any thing or things – on the actual existence of experts of Logic, for instance.[7]

When speaking on the "logic of God", we can understand the logic of our reasoning about God or the logic as it is supposedly employed by God (in the sense alluded to above). It is rather obvious, at least for believers in God, that we can infer something about God's logic in the latter meaning, from how the logic operates in the world created by Him. In the present essay, my strategy is to use this narrow window through which we can grasp some glimpses of "Gods ways of thinking".

There are strong reasons to believe that it is category theory that best displays the role of logic in the system of our mathematical and physical knowledge. It gives us a refreshingly new perspective on logic and its various applications, and could be a good starting point for our speculations concerning the "logic of God". Accordingly, we start, in section 2, with a quick look at main characteristics of category theory, in particular its relation to logic. New categorical perspective can successfully be applied to physics, as we briefly present in section 3. This process of "categorification of physics" clearly displays logical underpinnings of physical theories. The fact that logic can change from theory to theory, or from a level to a meta-level, poses the question of the existence of "superlogic" to which all other logics would somehow be subordinated. This question is discussed in section 4. The fact that it remains unanswered forces us to face the problem of plurality of logics.

Usually, it was tacitly assumed that the role of "superlogic" was played by classical logic with its non-contradiction law as the most obvious tautology. In sec-

[7] J. Leslie, R.L. Kuhn (eds.), *The Mystery of Existence. Why Is There Anything At All?* Wiley-Blackwell, 2013, p. 3 (from "General Introduction").

tion 5, we discuss paraconsistent logic as an example of a logical system in which contradictions are allowed, albeit under the condition that they do not make the system to explode, i.e. that they do not spill over the whole system. Such logic is an internal logic in categories called cotopoi (or complement topoi). In section 6, we refer to some theological discussions, both present and from the past, that associate "God's logic" with classical logic, in particular with the non-contradiction principle. However, the analyses carried out in the present essay teach us that the non-contradiction principle should not be absolutized. The only thing we can, with some certainty, assert on "God's logic" is that it not an exploding logic, i.e. that it is not an "anything goes non-logic". God is a Source-of-All-Rationality but His rationality need not to conform to our standards of what is rational. This "principle of logical apophaticism" is formulated and briefly discussed in section 7.

In the history of theology there is known at least one attempt to reconstruct the "process of God's thinking". I have in mind Leibniz's idea of God's selecting the best world to be created from among all possible worlds. In section 8, I suggest which modifications Leibniz would have introduced in his reconstruction, if he knew present developments in categorical logic.

The World of Categories

A category consists of a collection of objects:, x, y, z…, and a collection of morphisms (called also arrows) between objects. If x and y are objects, an arrow f from x to y is denoted by $f: x \rightarrow y$; x is called the source of f, a y its target. Two arrows f and g can be composed into $g \circ f$, if the target of f coincides with the source of g. Arrows have a right and left units, and the composition of arrows is associative.

It is surprising that such a seemingly simple structure could have such a powerful impact on the entire mathematics, logic, and potentially also on philosophy.

An archetypal example of category is the category of sets as objects, and maps between sets as morphisms (this category is denoted by SET). However, the SET category differs from the standard formulation of set theory in that it puts major emphasis on morphisms than on objects. This is typical for category theory. It is even possible to formulate this theory entirely in terms of morphisms with no mention of objects.[8]

Category theory exhibits a great tendency to universality. Most important concepts of category theory are defined in such a way that they pick up properties of properties rather than just properties of mathematical structures. In this way, they find their implementation in various areas of mathematics, sometimes conceptually very distant from each other. This allows one to detect dependencies (many of them unknown so far) between different mathematical theories. Category theory permeates all of the contemporary mathematics. Almost all modern expositions of mathematical theories begin with an at least sketchy presentation of their categorical aspects.

Another important characteristics of category theory, not independent of previously mentioned ones, is its exceptional unifying power. It ingeniously combines mathematical structures and logic into its own internal architecture; in the following sense: in principle, any mathematical structure can be internalised in a category having rich enough structure to serve the purpose. We say that a given mathematical structure is internalised in a given category if this structure can be implemented into (or modelled by) the logical pattern of arrows of a given category. Roughly speaking, system of arrows of a given category forms an "algebra of arrows" which can be interpreted as a certain logic (like Boolean algebra can be interpreted as classical logic).

The relationship between category theory and logic is a huge topic, and it is relatively well understood only with regard to a class of categories called topoi (or toposes). The concept of topos is an interesting and efficient tool in categorical reasoning. Although a topos can have objects that are very different and much richer than sets, the abstract properties of such a topos are, in principle, the same as those of ordinary sets, and many set theoretic constructions can almost literally by transferred to them.

In topoi, the aforementioned unifying power of categories assumes especially transparent form. On the one hand, they can be interpreted geometrically as generalised spaces, on the other hand, they possess their own internal logic. One could say that geometry provides a semantic aspect for the internal logic of a giv-

[8] M. Heller, *Category Free Category Theory and Its Philosophical Implications*, "Logic and Logical Philosophy" 2016, No. 25, pp. 447–459, DOI: 10.12775/LLP.2016.015

en topos.[9] It turns out that the internal logic of topoi is the intuitionistic logic. This means that the law of excluded middle (and consequently that of double negation) and the axiom of choice are forbidden.

We should also mention the so-called higher category theory. The idea is the following. An ordinary category (1-category) consists of objects and morphisms. In a 2-category, morphisms of the ordinary category are objects, and morphisms between morphisms are its morphisms (2-morphisms). Continuing in this way, we obtain n-objects (as (n-1)-morphisms) and n-morphisms (as morphisms between (n-1)-morphisms), and consequently an n-category. This construction is not as simple as it looks at the first glance. Difficulties increase with each step along the ladder of generalisations, even more so that at some steps generalisations can go in various directions. In this essay, we shall, in principle, stick to 1-categories with only marginal mentions of other possibilities.

Categories are not only worlds in themselves, each of them with its own internal logic and geometry, which can be ontologically interpreted, they also constitute an *ensemble* of worlds, the internal logic of which remains strictly related to logics of its constituent sub-worlds (subcategories). In this conceptual context, the tacitly assumed idea that it is classical logic that provides an "ontological framework" for the entire reality, looks highly suspect.

Categorical Physics

The above "philosophy of categories" is not only a mathematicians' game; it finds applications to physical theories as well. They assume two forms. The first form is more general and leads to the program of "categorification of physics"; the second, more radical, consists in reformulating (or reinterpreting) physical theories with the help of categorical tools.

Categorification, in general, is the process of finding categorical counterparts of set-theoretic concepts. This is done by replacing sets with categories, functions with functors, and equations that functions must satisfy by natural isomorphisms between functors satisfying, if necessary, some additional conditions.

[9] This statement can be given a precise form. Semantics in question is the so-called Kripke semantics for intuitionistic logic; see R. Goldblatt, *Topoi. The Categorial Analysis of Logic*, Dover – Mineola – New York 2006, pp. 256–263.

Since physical theories are founded on set based mathematics, they can also be subject to the categorification process.[10]

As noticed by Baez and Dolan, "many deep-sounding results in mathematics are just categorifications of facts we learned in high school".[11] In the standard mathematical practice we unwittingly "decategorify" mathematical structures by treating categories as just sets and functors as just functions between sets. In this way, we erase some information even before we discover it. Restorung this information, i.e. performing categorification, is a difficult and non-unequivocal task. If this process is applied to physical theories, one can hope to gain hidden information that could be indispensable to solve some unsolved problems or to reinterpret badly understood theories.

This leads us to the second form of category theory applications to physics: one does not try to "translate" set theoretical structures into their categorical counterparts, but simply employs categorical concepts and categorical toolkit to physical theories, models or concrete problems. There are many such attempts. Perhaps the best developed one is the program of reformulating quantum mechanics with the help of topos theory elaborated by Isham's group[12] and Moerdijk's team[13]. But there are also applications to general relativity and cosmology.[14]

From the discovery of quantum logic, which essentially differs from classical logic,[15] the tacit assumption that the latter has the universal validity for the entire physics had to be subject to revision. It is rather evident that macroscopic physical theories do not require anything beyond rules of classical logic, but nothing like that can with certainty be asserted with respect to submicroscopic physics.

[10] See: J.C. Baez, J. Dolan, *Categorification*, arXiv:math/9802029[math.QA].

[11] Ibid.

[12] A. Döring, C.J. Isham, *A Topos Foundation for Theories of Physics*, I. arXiv:quant-ph/0703060, II. arXiv: quant-ph/0703061, III. arXiv:quant-ph/07030640, IV. arXiv:quant-ph/0703066. 242

[13] C. Heunen, N.P. Landsman, B. Spitters, *Bohrifcation*, in: *Deep Beauty: Understanding the Quantum World through Mathematical Innovation*, H. Halvorson (ed.), Cambridge University Press, Cambridge 2011, pp. 271–313, arXiv:0909.3468.

[14] See for instance: A.K. Guts; Y. B. Grinkevich, *Toposes in General Relativity*, arXiv:gr-qc/9610073; C. Heunen, N.P. Landsman, B. Spitters, *The Principle of General Tovariance*, in: *International Fall Workshop on Geometry and Physics XVI, AIP Conference Proceedings*, 2008, Vol. 1023, pp. 93–102, DOI: 10.1063/1.2958182, http://homepages.inf.ed.ac.uk/cheunen/publications/2008/tovariance/tovariance.pdf; M. Heller, J. Król, *Synthetic Approach to the Singularity Problem*, arXiv:1607.08264[gr-qc].

[15] The classical paper is: G. Birkhoff, J. von Neumann, *The Logic of Quantum Mechanics*, "Annals of Mathematics" 1936, Vol. 37, pp. 823–843.

A hypothesis has been put forward that logic could be a "physical variable" in the sense that it changes from theory to theory depending on which level (macro, micro, submicro…) a given theory operates.[16]

Is there a Universal Logic?

In the light of the above the question "Is there a universal logic?" seems important both from the philosophical and the theological points of views. If there exists a "superlogic" – a logic which is "meta" with respect to all other logics, or to which all other logics are somehow subordinated – then we could justifiably interpret such a superlogic as related to the ontology of reality or even as telling us something about the "logic of God". If this is not the case, we would have, both in philosophy and theology, face the difficult problem of "logical pluralism". So far the answer to this question is not known, and the problem itself is subject to an intense debate.[17]

In fact, we should distinguish between two different questions: one addressed to the logic of our language and our ways of reasoning, and second addressed to the "logic of the world". Roughly speaking, with the first question usually logicians are busy, the second is presupposed by physical theories. However, we should further distinguish, after Shapiro,[18] "the logic of a given mathematical [or physical] theory and the underlying logic of the philosophical discourse used to discuss this theory", and remember that the "philosophical discourse used to discuss a theory" belongs to our ways of reasoning rather than to the logic of the

[16] J. Król, *A Model for Spacetime: the Role of Interpretation in Some Grothendieck Topoi*, "Foundations of Physics" 2006, Vol. 36(7), pp. 1070–1098.; M. Heller, J. Król, *How Logic Interacts with Geometry: Infinitesimal Curvature of Categorical Spaces*, arXiv:03099v1[math.DG].

[17] There is another „foundational problem" in category theory that could be relevant for our discussion, namely: Can category theory be formulated without presupposing the concept of set? The question is still subject to debate (see, for instance, A. Blass, *The Interaction between Category Theory and Set Theory*, "Contemporary Mathematics" 1984, Vol. 30, pp. 5–29; F. A. Muller, *Sets, Classes, Categories*, "British Journal for the Philosophy of Science" 2001, Vol. 52, pp. 539–573). A naïve theological counterpart of this question could be: Is God necessarily thinking in terms of sets?

[18] S. Shapiro, *Varieties of Logic*, Oxford University Press, Oxford, 2014, s. 176.

world. Categorical logic[19] can appear in both guises: as our logic and as the logic of the world.

In what follows, we shall put aside the universality question as it is referred to what we have called "our logic",[20] and focus on the "logic of the world" or, more precisely, on the logic as it is presupposed by various mathematical and physical theories. For the sake of concreteness, we shall limit our analysis to the problem as it is seen in the context of category theory.

From the fact that the question related to the existence of superlogic (mentioned above) seem helpless and from the entire foregoing discussion it is evident that a kind of logical pluralism is unavoidable. Some hopes were initially associated with the proposal of a category of categories,[21] but it turned out that it admits several formulations, the weaker of which are "workable" but useless as far as our question is concerned, and the stronger ones are inconclusive and involved in ambiguities.

Logical pluralism can have profound consequences as far as philosophical interpretations are concerned. Let us take an example from philosophy of mathematics. According to J.L. Bell's proposal,[22] the theory of sets, on which the present philosophy of mathematics is founded, should be replaced by the theory of topoi, in fact by those topoi that are equipped with the natural numbers object (such topoi are called by Bell local frames). Mathematical concepts are to be defined not absolutely, but rather with respect to a given local frame. In this way, they lose their "absolute meaning" they possess in the set theoretic interpretation, and have only the meaning with respect to a given local frame. Since every topos has its own internal logic, it becomes a varying element of the whole strategy, changing from one local frame to another local frame. One could say that logic becomes a "mathematical variable".

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19 Here and below by categorical logic I do not understand this expression in its technical meaning (see, S. MacLane, I. Moerdijk, *Sheaves in Geometry and Logic*, Springer, New York–Berlin–Heidelberg 1992, p. 526), but rather in a broad sense as a totality of categorical results and methods related to logic.

20 The interested reader should be referred to: B. Czernecka-Rej, *Pluralizm w logice*, Wydawnictwo KUL, Lublin 2014 (in Polish); ample literature is quoted in this book.

21 The original proposal was put forward by F. W. Lawvere, *The Category of Categories as a Foundation for Mathematics*, in: *Proceedings of the Conference on Categorical Algebra*, Springer, La Jolla, New York, 1966, pp. 1–21.

22 J.L., Bell, *From Absolute to Local Mathematics*, "Synthese" 1989, Vol. 6,9 pp. 409–426.

It is rather obvious that Bell's approach can be easily adapted to philosophy of physics, and it is more than enough to seriously face the possibility that also in philosophy in general, and in the philosophy of God in particular, something similar should be taken into account.

An Example

To see the extent of consequences the "variability of logic" can have for philosophy and theology, let us consider an example. Perhaps the most "paradigmatic" example of classical logic is the non-contradiction principle (see Introduction). Let us look at it and its limitations in the framework of category theory.

As it is well known, the set $\mathcal{P}(D)$ of all subsets of a set D has the Boole algebra structure and is a model of the classical propositional calculus. Since conjunction corresponds to intersection of subsets, alternative to their sum, and negation to the complement of a given subset, the non-contradiction principle is written as

$$A \cap \overline{A} = \emptyset$$

where $A \in \mathcal{P}(D)$. This principle is not a tautology in the so-called paraconsistent logic. The model of this logic is the set $Cl(X)$ of all closed subsets of a topological space X. It has the structure of a co-Heyting algebra (called also Brouwer algebra). In this case, just as above, conjunction corresponds to intersection of closed subsets, and alternative to their sum, but the complement of the closed set is not closed, therefore negation has to be defined as the closure of the complement of the closed subset (corresponding to a proposition p). Thus, the contradiction principle is written as

$$W_p \cap \overline{(W_p)^c}$$

where $W_p \in Cl(X)$. In classical logic the Duns Scotus law holds true stating that from contradiction anything follows (*ex contradictione quodlibet*), and the system is automatically overfilled (one says also exploded). However, in paraconsistent logic, this does not happen as it can be seen from the following example.

Let us consider the real line $\mathbb{R}$ with its natural topology as our topological space X, and let the closed subset $[0, 1]$ corresponds to a proposition p. We obviously have

$$Wp \cap \overline{(W_p)}^c = \{0\} \cup \{1\}$$

As it can be seen, the contradiction does not spill over the entire system since it is imprisoned in the boundary set {0} ∪ {0}. The conclusion is that we should be not afraid of contradiction but rather of overfilling the system.

It turns out that paraconsistent propositional calculus is dual with respect to intuitionistic propositional calculus, and as the latter is typical for topoi, the former is typical for cotopoi (called also complement topoi).[23] This shows that paraconsistent logic is not only the logic of a discussion, in which utterances of two disputants can be contradictory with each other but contradictions are not allowed to spill over the views of each of them separately, but can also be the internal logic of an abstract world modelled by a cotopos.

"The Lord of Non-Contradiction"

Let us now confront the above example with the following apodictic statement:

> Logic is irreplaceable. It is not an arbitrary tautology, a useful framework among others. Various systems of cataloguing books in libraries are possible, and several are equally convenient… But there is no substitute for the law of contradiction. If dog is equivalent of not-dog, and if 2 = 3 = 4, […] zoology and mathematics disappear…[24]

The author of these words claims that "the law of contradiction, is neither prior to neither subsequent to God's activity" since "God and logic are one and the same first principle".[25]

The title of the present section is borrowed from another article devoted to the question of "God's logic".[26] The authors are interested in what they call "substantial metaphysical relationship between the laws of logic and the existence of God" with the arrow of dependence going from God to the laws of logic; that is to say,

23 See L. Estrada-González, *Complement Topoi and Dual Intuitionistic Logic*, "Australian Journal of Logic" 2010, Vol. 9, pp. 26–44.

24 G. H., Clark, *God and Logic*, "The Trinity Review" 1980, December–November, p. 7; http://www.trinityfoundation.org/journal.php?id=16

25 *Ibid.*, p. 3.

26 J.N. Anderson, G. Welty, *The Lord of Non-Contradiction: An Argument for God from Logic*, "Philosophia Christi" 2011, Vol. 13, Issue 2, pp. 321–338.

they argue that "there are laws of logic *because* God exists; indeed, there are laws of logic *only* because God exists". Their argument runs as follows. (1) The laws of logic are truths; (2) they are truths about truths; (3) they are necessary truth (e.g. "the Law of Non-Contradiction is true not only in the actual world, but also in every possible world"); (4) they really exist; (5) they necessarily exist (this follows from (3): "if the laws of logic are *true* in every possible world, it follows sensibly that they *exist* in every possible world"); (6) they are non-physical (it does not make "any sense to ascribe *physical properties* to the Law of Non-Contradiction, such as mass or velocity or electric charge. It simply isn't that *kind* of thing"); (7) they are thoughts; (8) they are Divine Thoughts. The authors underline that the law of non-contradiction is for them only a typical example, but their argument does not require acceptance of any particular law of logic or any particular logical system; it is enough that some laws of logic do exist. "While there may be debates over *which* laws of logic hold, there is no serious debate over whether there *are* laws of logic".

This short summary does not make full justice to the authors' argument. It goes without saying that it is much more elaborated. Anyway, it is not my intention to enter into polemic with them; I quote their views only to illustrate one of the typical standpoints in the question about God and logic. In fact, it is not far from Leibniz's view who claimed, as succinctly summarized by Amos Funkenstein, that "logical necessity is grounded on the principle of noncontradiction only, under which God's will and even his thought are subsumed".[27]

The whole question goes back to the Medieval dispute concerning God's omnipotence. Petrus Damiani put no limits on God's power asserting that God could do everything he wished, even logically impossible things or, for instance, reverse past events ("and have Rome not to be founded"). Anselm of Canterbury strongly objected. In his view "a God that can create contradictions could also annihilate himself together with his omnipotence; a God that may be thought of as non-existing cannot be an *ens necessarium* either. God's will is at least bound by the principle of noncontradiction".[28]

[27] A. Funkenstein, *Theology and the Scientific Imagination from Middle Ages to the Seventeenth Century*, Princeton University Press, Princeton, 1986, p. 118.

[28] *Ibid.*, p. 128.

Logical Apophaticism

What lesson follows for the "logic of God" from the categorical approach to logic? The straightforward conclusion is that the principle of non-contradiction should not be absolutized. As we have seen, we should be afraid of exploding the system rather than of a contradiction operating in it. Paraconsistency could not only be innocuous, but even creative and enriching. To translate old theological antinomies (like "God creating a rock so heavy that He could not lift it" or some of Anselm's antinomies) into paraconsistent type of logic, could be a good exercise in "theological logic", but nothing more than that. Claiming that God in His thinking is obliged to use one of our logical systems, be it classical or categorical, would result in another anthropomorphic belief. Judeo-Christian tradition always regarded God as a Supreme Rationality that transcends all human patterns of thinking.

Nevertheless, the discovery of categorical logic is not irrelevant for theological analyses. It has given us a powerful warning that indiscriminate use of classical logic (with its non-contradiction principle) in reference to God (like the examples mentioned in the preceding section) could lead to false conclusions and hasty simplifications.

The only thing we can assert about God's logic with reasonable certainty is that it is not a logic in which "anything goes", that is to say, that it is not an "exploding logic". In Judeo-Christian tradition, God is the Source-of-All-Rationality, but it need not to conform to our standards of what is rational and what is not. I would call the above statement the *principle of logical apophaticism*. To claim something more would be to pretend we know what, in fact, we do not know. This principle is a kind of *docta ignorantia* since it is motivated by our "learned knowledge" of modern logic, including categorical logic, which has warned us not to absolutize any logical system, classical logic in particular. This sort of *ignorantia* is certainly not opening a gate for theological irrationalism; it is only the consequence of being aware of our limitations. And such an awareness is but a necessary condition of a sound rationality.

Instead of Conclusions: Another Example

Can we find, in the history of philosophy or theology, an attempt to reconstruct God's reasoning referring to some particular problem? There exists at least one such attempt. I have in mind Leibniz's idea of God's selecting the best world from among all possible worlds. The reconstruction of God's deliberations runs as follows. God decides to create the best possible world. To do so He scrutinizes all possibilities, and looks for the world in which the amount of goodness is maximized. However, very often ethical requirements clash with each other, therefore not all goods could coexist at the same time. This is why there are not individual goods that concur with each other, but rather all their possible combinations. This combination deserves to be selected by God that is the best of all without being involved in contradictions.

If Leibniz knew modern logic, he would have made the choice even more difficult for God. In pondering all possibilities, He would have to allow for contradictions, watching only that they would not spill over the entire selected world.

Leibniz was aware that this optimistic view would be subject to a strong criticism. He explicitly states that "[…] the world, especially if we consider the government of human race, seems rather a confused chaos than anything directed by a supreme wisdom", and he explains that such a judgement is the result of our narrow perspective, since "we know a very small part of eternity which is immeasurable in its extent".[29] How can we judge infinity, knowing only a single point? This is why the logical apophaticism is more than justified.

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Summary

When speaking on the "logic of God", we can understand the logic of our reasoning about God or the logic as it is supposedly employed by God. It is rather obvious, at least for believers in God, that we can infer something about God's logic in the latter meaning, from how the logic operates in the world created by Him. In the present essay, my strategy is to use this narrow window through which we can grasp some glimpses of "God's ways of thinking".

There are strong reasons to believe that it is category theory that best displays the role of logic in the system of our mathematical and physical knowledge. It gives us a refreshingly new perspective on logic and its various applications, and could be a good starting point for our speculations concerning the "logic of God". A quick look at category theory and its applications to physics shows that logic can change from theory to theory, or from level to meta-level. This poses the question of the existence of "superlogic" to which all other logics would somehow be subordinated. The fact that this question remains unanswered forces us to face the problem of plurality of logics.

Usually, it is tacitly assumed that the role of "superlogic" is played by classical logic with its non-contradiction law as the most obvious tautology. We briefly discuss paraconsistent logic as an example of a logical system in which contradictions are allowed, albeit under the condition that they do not make the system to explode, i.e. that they do not spill over the whole system. Such logic is an internal logic in categories called cotopoi (or complement topoi). I refer to some theological discussions, both present and from the past, that associate "God's logic" with classical logic, in particular with the non-contradiction principle. However, we argue that this principle should not be absolutized. The only thing we can, with some certainty, assert on "God's logic" is that it is not an exploding logic, i.e. that it is not an "anything goes logic". God is a Source-of-All-Rationality but His rationality need not to conform to our standards of what is rational. This "principle of logical apophaticism" is formulated and briefly discussed.

In the history of theology at least one attempt is known to reconstruct the "process of God's thinking", namely Leibniz's idea of God's selecting the best world to be created from among all possible worlds. Some modifications are suggested which we believe Leibniz would have introduced in his reconstruction, if he knew present developments in categorical logic.

Key words: category theory, paraconsistent logic, logical pluralism, theology and logic, logical apophaticism

Streszczenie

Logika Boga

Mówiąc o „logice Boga", możemy mieć na myśli logikę naszego myślenia o Bogu albo logikę, jaką w naszym wyobrażeniu posługuje się Bóg. Jest dość oczywiste, przynajmniej dla wierzących, że to i owo na temat logiki Boga w tym drugim znaczeniu możemy wywnioskować z logiki obowiązującej w stworzonym przez Niego świecie. W tym eseju stosuję właśnie tę strategię, by zidentyfikować niektóre przebłyski „Boskiego sposobu myślenia".

Istnieją dobre powody, by przyjąć, że rolę logiki w systemie naszej matematycznej i fizycznej wiedzy najlepiej obrazuje teoria kategorii. Daje nam ona nowe, świeże spojrzenie na logikę i jej różne zastosowania, i może być dobrym punktem wyjścia dla badań nad „logiką Boga". Pobieżne nawet przyjrzenie się zastosowaniom teorii kategorii w fizyce pozwala się przekonać, że logika może się zmieniać przy przejściu od teorii do teorii i z jednego poziomu ogólności na drugi. Nasuwa się pytanie o istnienie „superlogiki", której w jakimś sensie podlegałyby wszystkie logiki niższego rzędu. Fakt, że nie potrafimy na to pytanie odpowiedzieć, stawia nas w obliczu problemu logicznego pluralizmu.
Zwykle przyjmuje się milcząco, że rolę „superlogiki" odgrywa logika klasyczna z zasadą niesprzeczności jako najbardziej oczywistą tautologią. Artykuł omawia krótko logikę parakonsystentną jako przykład logiki, w której sprzeczności są dozwolone, choć jedynie pod warunkiem, że nie eksplodują, tzn., nie rozlewają się na całość systemu. Taka logika jest wewnętrzną logiką w kategoriach zwanych ko-toposami (*complement topoi*). Przywołuję niektóre teologiczne dyskusje, zarówno dawne, jak i współczesne, w których „logikę Boga" utożsamiano z logiką klasyczną, a zwłaszcza z zasadą niesprzeczności, starając się pokazać, że zasady tej jednak nie powinno się absolutyzować. Jedyne, co z jakąś pewnością możemy powiedzieć o „logice Boga", to że nie jest to logika bez żadnych reguł, w której wszystko jest dozwolone. Bóg jest Źródłem-Wszelkiej-Racjonalności, ale Jego racjonalność nie musi spełniać naszych standardów. Formułuję zatem i krótko omawiam tę „zasadę logicznej apofatyczności".
Historia teologii zna przynajmniej jedną próbę zrekonstruowania „procesu Boskiego namysłu" – wizję Leibniza, u którego Bóg ze wszystkich światów, które mógłby stworzyć, wybiera najlepszy. Sugeruję na koniec kilka zmian, które moim zdaniem Leibniz wprowadziłby do swojej rekonstrukcji, gdyby znał współczesną logikę kategorialną.

Słowa kluczowe: teoria kategorii, logika parakonsystentna, pluralizm logiczny, teologia a logika, logiczna apofatyczność