Edukacja Filozoficzna

70/2020

Dedicated to Józef Maria Bocheński

Wydział Filozofii Uniwersytetu Warszawskiego

Wydawca / Publisher

Uniwersytet Warszawski, Wydział Filozofii

ISSN 0860-3839

Nakład / Print run: 118 egz./copies

Adres redakcji / Address: ul. Krakowskie Przedmieście 3, p. 302, 00-927 Warszawa
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Czasopismo „Edukacja Filozoficzna" znajduje się w wykazie czasopism naukowych MEiN (20 punktów) / *Edukacja Filozoficzna* features in the Ministry of Education and Science's index of scholarly journals (20 points).

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Józef Maria Bocheński: An Inspiring Scholar

A priest and a member of the Dominican Order, a soldier who fought in the war of 1920 in which Poland defended Europe against Soviet Russia, and in the Second World War, including in the Battle of Monte Cassino, a counsellor, and even… a negotiator,[1] finally: a lecturer and a prominent scholar – a logician, philosopher and the first Sovietologist.

Józef Maria[2] Bocheński (1902–1995) played many roles in his long life. However, various spheres of his activity overlapped, giving impressive results which inspired both scholars and people who used them in their practice: his ethics for soldiers based on his war experience and philosophical method, the enquiries in which he applied modern logic to theology and religion, or his engagement in politics within which he used his deep knowledge of communism to support trials and processes aimed at banning communist parties in many countries, to name but a few examples.

Among scholars from all over the world he is known for his: logic of religion, theory of analogy, logical analysis of Aquinas's five ways, and many works on the history of logic. He is also appreciated for his ethical investigations, theory of superstitions and comparison of patriotism and nationalism. His ideas and research results, as well as his methodological approach, are an inspiration for many. At the same time, he often aroused strong emotions, due to his uncompro-

[1] Vividly depicted in the film *Negocjator* [Negotiator], produced by Teatr Telewizji [Television Theatre], URL: https://vod.tvp.pl/website/negocjator,46547106.

[2] He was baptized Józef Franciszek Emanuel, and in the Dominican Order he took the names Innocenty Maria; however, he is traditionally known as "Józef Maria Bocheński."

mising statements and behaviour. But also in this way he encouraged others to think deeper and not to be content with shallow slogans.

In this issue, we would like to commemorate and honour this outstanding thinker, at the end of Józef Maria Bocheński's Year, officially celebrated in Poland in 2020 – on the 25th anniversary of his death.

We have collected papers which refer to some of his findings. They are divided into two groups. The first one consists of articles which discuss several aspects of Father Bocheński's scholarly enquiries. Three of these papers were presented at the conference "The Logical Structure of the World: An Axiological Vision of Patriotism. On the XXV Anniversary of the Death of J.M. Bocheński," held online on 15–16 October 2020 at the Cardinal Wyszyński University in Warsaw.

The second group of articles includes papers which are inspired by Bocheński's thought and which implement some of his ideas and methodological approaches. All the articles are preceded by an introduction written by one of his disciples, Dr Jan Parys, highlighting the most important biographical facts which shaped the scholar's intellectual profile, and the way he developed one of his main ideas, that is, the compatibility of faith and reason.

Initially, we had also considered publishing a bibliography of Father Bocheński's works at the end of this volume, but we have found well-prepared bibliographies available online, and so it seemed unnecessary to reproduce them here. We refer the reader to a very extensive one, which lists not only the publications of Father Bocheński but also secondary literature and sources concerning his life and work: https://pbw.org.pl/przemysl-2,55/jozef-innocenty-maria-bochenski-1902-1995-bibliografia-podmiotowo-przedmiotowa,12276.

We would like to thank the main organizer of the above-mentioned conference, Prof. Kordula Świętorzecka, for inviting the speakers to contribute to this volume. We are also very grateful to our reviewers, who assessed the articles and helped to improve them, and finally: to all the authors for their great contributions and their efforts. We believe that this issue will be a worthy part of the celebrations of Józef Maria Bocheński's Year and that it will serve to promote the work and ideas of this scholar.

Dear Readers, we also hope that you will find this issue interesting and inspiring. We wish you a pleasant reading.

The *Edukacja Filozoficzna* Editorial Team

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Bocheński: Science and Faith[1]

Jan Parys

Józef Maria Bocheński, as a logician, tried to cure society of superstitions and, as a priest, he tried to heal the souls of people lost in the world. He died in February of 1995, at the age of 93. As a believer, he bequeathed his earthly remains to the University of Fribourg, so that medical students would not be short of material to study. Needless to say, this part of his will evoked some indignation. In Bocheński's opinion, the clause was merely a reiteration of his strong belief in life after death. We are interested in Bocheński's mortal life as a philosopher. In order to understand who he was and why he thought the way that he did, it is first worth taking a look at Polish philosophy in the 20th century.

The Lvov-Warsaw School

When talking about Polish intellectual advances, we usually mention Nicolaus Copernicus, Maria Skłodowska-Curie, and Ludwik Hirszfeld. Meanwhile, in the 20th century, Polish scholars contributed to the world of science in the field of philosophy. This contribution is comprised of the achievements of a group of logicians and philosophers known as the Lvov-Warsaw School. It is thanks to this school that in the 20th century Poland became a philosophical power.

[1] This introductory article was originally published in Polish: J. Parys, *Nauka i wiara: na 20. rocznicę śmierci Józefa M. Bocheńskiego*, "Arcana" 2015, Vol. 2, pp. 145–155.

The term "Lvov-Warsaw School" was first used by Kazimierz Ajdukiewicz at a philosophical congress in Paris in 1935. Since then, the achievements of Polish logicians have gained international recognition. No wonder that during the construction of the University of Warsaw Library, when it was decided that statues of four world-famous Polish scholars would be placed by the entrance, philosophers from this school were chosen, namely Kazimierz Twardowski, Alfred Tarski, Jan Łukasiewicz, and Stanisław Leśniewski.

It is difficult to explain how, at the beginning of the 20th century, Poles, who up until then had not had any strong or unique philosophical currents of their own, came to be the founders of a world-famous philosophical school. How much of it was due to the will of God, and how much was coincidence? Keeping in mind, of course, that God does not act directly but through people.

In the case of the school, it all began with Kazimierz Twardowski (1866–1938), who taught philosophy at Lvov University from 1895 to 1938. One might say that he was lucky to have talented students, as over 30 of them went on to become professors. In 1938, almost 80 scholars were part of the Lvov-Warsaw School. The members of the school differed in their views on many issues, but they shared something that could be called an analytical orientation. Today, the third generation of Twardowski's students is active in Poland; we refer to them as proponents of analytical philosophy.

This school was not just made up of logicians and philosophers. Twardowski had a broader influence on scholars in many fields. His concepts of science were adopted by physicists such as Zygmunt Zawirski and Czesław Białobrzeski, mathematicians such as Stanisław Jaśkowski and Andrzej Mostowski, literary scholars such as Zygmunt Łempicki and Stanisław Łempicki, sociologists such as Maria Ossowska and Stanisław Ossowski, psychologists such as Władysław Witwicki and Eugeniusz Geblewicz, and lawyers such as Czesław Znamierowski. In short, Twardowski influenced not only philosophy, but the entire Polish intellectual scene. What is more, his influence continues to this day. Several generations of students have already used Tadeusz Kotarbiński's logic and methodology textbook, Ajdukiewicz's *Zagadnienia i kierunki filozofii* [Problems and Theories of Philosophy], and Władysław Tatarkiewicz's *Historia filozofii* [History of Philosophy]. Therefore, it can be said without exaggeration that the works of the philosophers from this school have influenced the very way Polish intellectuals think.

It is worth adding that the views of the members of this school varied on politics and religion, for example:

- Tadeusz Czeżowski was the director in the Ministry of Education in the Second Polish Republic; during the war he hid Jewish people from the Germans;
- Łukasiewicz was the Minister of Education in Ignacy Paderewski's government in the Second Polish Republic;
- Kotarbiński and Ajdukiewicz had views typical of the left-leaning pre-war intelligentsia; in the Polish People's Republic, Kotarbiński was the President of the Polish Academy of Sciences, and Ajdukiewicz was the dean of the Adam Mickiewicz University in Poznań;
- the group of Catholic thinkers within the school consisted of Father Jan Salamucha and Father Bocheński, as well as professors Jan Łukasiewicz, Bolesław Sobociński, and Jan Drewnowski.

Twardowski was not only a scholar but also an outstanding educator; he knew how to teach and how to organize academic life. He began to give lectures when Poland was still partitioned, when Lvov belonged to Galicia, a part of the Austrian empire of the Habsburgs. In 1904 he founded the Polish Philosophical Society in Lvov. In 1911, he also founded *Ruch Filozoficzny* [Philosophical Movement], a journal that exists to this day. He believed that through modern philosophy you could teach people to think properly, that is to say, with precision and without psychologizing. In his opinion, through the modern concept of science, we can and should change the way Polish people think.

What was the distinctive feature of this school? It is hard to say that there are any specific, distinguishing theses. As Bocheński emphasized in his essay *O filozofii analitycznej* [On Analytic Philosophy], the school is rather a shared research perspective, an orientation different from what had been previously encountered in science. The supporters of the school, even though they differed on many issues, believed that in scientific research one should above all follow these four principles or slogans:

- analysis – we are aware that the world is complicated, and so we reject great syntheses and conduct small analyses, which we use to build science;
- language – meaning one should express oneself clearly; philosophy, like any scientific discipline, is not a matter of playing with words or some lite-

rary description of the world; a man of science is someone who can explain his thoughts to others;
- logic – that is, scientific thinking must comply with the laws of logic, which is a type of universal language for every field of thought;
- objectivism – philosophy is to refrain from what is subjective; it is to help in the study of the world, and it is to guard reason; it is not about building an ideology or a vision; philosophy is not a worldview, it is not obliged to resolve moral dilemmas; philosophical analysis is to begin from the analysis of the world, not from the analysis of psychological human experiences.

What are the philosophers of the school famous for today, after several decades? I will list only some of their achievements:
- Łukasiewicz invented the Polish notation known as the Łukasiewicz notation; he also invented many-valued logic;
- Tarski invented a semantic definition of truth; he provided the definitions of logical investigation;
- Ajdukiewicz formulated the classification of reasonings;
- Leśniewski distinguished languages and metalanguages;
- Father Bocheński wrote the first history of logic from the perspective of contemporary logic; he built a logic of religion, proved the consistency of logic and religion, and he formulated the concept of analytical Thomism;
- Father Salamucha conducted a logical proof for the existence of God; he was a proponent of the use of modern logic in studying old questions posed in Christian philosophy, even those posed in the Middle Ages.

Each of these men is respected for what he wrote – clearly their works have stood the test of time. And time is the best judge for distinguishing what is fashionable from what is true and important in science.

Bocheński's Path to Faith and Science

How did Bocheński come to his faith? In his memoirs, he openly states that he was not always deeply religious. Like many young people, he wanted to partake in life. At university, he changed his major several times; he was no stranger to the joys of student life, and he devoted a great deal of time to being active in student associations. He took his time studying because he was looking for his path in

life – for a long time he was searching for an answer to one question: what ought one do in life in order for it to have meaning? Of course, that is a difficult question to work through, and, what is more, it is one that everyone has to answer for themselves. Bocheński's life changed when he met a distinguished Dominican scholar, Father Jacek Woroniecki. He became an authority figure for Bocheński and convinced him that since there was an economic, political, and moral crisis in Europe, one should seek support in what is lasting. And in Europe that was Christianity. At the time, Europe was beset by a global economic depression, crises of democratic governments, and the fashionable ideologies of Bolshevism and Fascism. Bocheński was not convinced by either of these ideologies. As he recalls, he entered the seminary and the Dominican novitiate with weak faith; he was almost an agnostic. It was more of a rational choice than one of faith, after he had grown disheartened by the world. He would come to conscious faith slowly while in the convent.

What was Bocheński's intellectual path like? Thanks to his well-to-do parents, he was able to study whatever he wanted and for as long as he wanted. First, he studied law in Lvov, then economics in Poznań. In 1926, he entered the seminary and a year later the Dominican novitiate in Poznań. He studied philosophy and theology, first in Kraków and Warsaw, then in Fribourg and Rome. He earned his doctorate in philosophy in Switzerland in 1932 and in theology in Rome in 1935; he obtained his habilitation in logic in Kraków at the Jagiellonian University in 1938. Before the war, he taught at the Angelicum in Rome. After the war, he was a professor at the Catholic university in Fribourg and the rector of that university from 1964 to 1966. He published over 100 books, some of which had many translations and circulations of up to a million copies. His most famous books are *A History of Formal Logic*, *Contemporary European Philosophy*, and *The Methods of Contemporary Thought*. It should come as no surprise that he received several honorary doctorates. He was also the first Polish philosopher to have his works published in the prestigious "Biblioteka Klasyków Filozofii" [Library of Philosophical Classics] series during his lifetime.

What is the relationship between faith and reason according to Bocheński, a philosopher who became a world-renowned Catholic scholar, who was not ashamed to appear in his monastic garb at the most important international congresses in order to emphasize that there is no contradiction between science and faith?

For many centuries, faith and science were two spheres of life that developed independently of one another. For a long time, these areas were presumed to be in conflict. The fact that for many decades the Church did not recognize Copernicus's theory often serves as an example of that. It even condemned supporters of that theory, such as the Italian philosopher Giordano Bruno. At times, condemnation meant being burnt at the stake. A statue of Giordano Bruno, which we can now see in Rome, in Trastevere, stands at the place of his death. He lost his life because he opposed the contemporaneous view of the world supported by, among others, the Church. Those times are long gone. We have witnessed a change in the attitude of the Church towards science.

Bocheński's Thesis

According to Father Bocheński, when we consider the relationship between faith and reason, it is worth starting with a reflection on the situation of our epoch, on the intellectual climate that surrounds us. Faith concerns God, the world, and the values we are to live by. Our epoch is characterized by haste, change, and the improvement of everything. Meanwhile, for believers, the truths of faith are as important today as they were in the times of Christ. These values are not subject to fashion, they do not need to be replaced or changed in the way that you might need to replace an old fridge. One just needs to understand them in changing times. Undoubtedly, conclusions must be drawn from the fact that nowadays the majority of society is educated, that we live amid new problems and new schools of thought, and that, for example, atheists live among us. However, that only means that the oldest truths of faith must be spoken about in a different language – those old truths must be related to the modern world that we perceive through the prism of science and technology.

In Europe's history, there have been scholars and thinkers that have tried to separate science from classical philosophy and from faith. That was especially the case during the period of so-called radical positivism at the end of the 19th and the beginning of the 20th century. However, it soon turned out that such an attitude leads to the amputation of many problems of the humanities, that one then loses the possibility of metaphysical reflection and is reduced to a human being merely reacting to sociological and biological conditions. Today we know that

the period of naive scientism that was so hostile to religion, popular at the turn of the 19th and the 20th centuries, has passed. Today we know that science does not have all the answers, that most natural scientists reject materialism. An outstanding physicist, the discoverer of the uncertainty principle, Werner Heisenberg, often referred to Plato and Descartes. Bocheński stated outright that there is no conflict between science and faith, and if there are contradictions therein, they result from faults in our minds. The stronger our faith is, the more we should trust science. A true believer has no doubts and does not see any contradictions in the world, because – since the world is logically constructed – we only have problems that we have not yet managed to understand and solve. Going even further, one may say that most philosophers, especially those of the 20th century, are Platonists who admit that there is no possibility for serious reflection on the world without the Absolute, without recognizing that there is an ideal world independent of our minds.

Even Aristotle wrote in *Metaphysics* about the need for theology, that is, a separate science of God. St Thomas Aquinas wrote about the need for *doctrina sacra*. A Polish philosopher from the 19th century, Bronisław Trentowski, postulated a Polish term for theology. He proposed that this field be called *bożyca* (which roughly translates to *godology*), as it teaches about "Bóg" (God). Today, disputes between philosophers regard mainly the nature of God and not his existence. The only known exception, a 20th-century philosopher who denied the existence of God, was Jean-Paul Sartre.

What do I mean by Christian intellectual reflection? Father Bocheński answered this question in the following way: it is not about the fact that the writer is a believer. We can imagine a nonbeliever writing an earnest study on the Bible, on Christ, and on the history of the Church. Hence, it is not about the writer, but about the content of what is written. Reflection is not Christian because it concerns the problems of Christianity. First and foremost, Christian reflection is distinguished by its being developed within the framework of a particular worldview.

Bocheński emphasized that faith has two aspects. On the one hand, faith is a state of mind, an act of accepting certain sentences. On the other hand, faith is what we believe in. And we believe in certain sentences that Catholics call the *credo*. Bocheński thought that most people did not experience God directly – only the disciples of Christ, who knew him, as well as prophets and saints had this experience. The common man rather encounters the word of God through

Scripture. In prayer we address God, but there is no dialogue. We adopt the Christian worldview through an act of will, and it does not need to be justified with methods considered to be scientific, as faith is faith and we do not need to prove its truth. That being said, we can and should understand and analyze what we believe in.

The following sentence is key for Bocheński's approach: "There is no proof for the truths of faith, but I must believe what I believe in." Bocheński never claimed that we come to know God solely through reasoning, without the help of revelation. However, he maintained that rational knowledge about God is possible. One might ask what is so original about that, given that a few centuries earlier the First Vatican Council condemned fideism, emphasizing that one cannot come to know God independently of reason. The thing is that many generations forgot about that decision of the Council, and theology often developed in isolation from logic and the achievements of science.

I believe that Bocheński's contribution was the development of the argumentation for this position of the Council, formulated so long ago. When writing *The Logic of Religion* in 1965, he pointed out the usefulness of formal logic in studying the structure of religious statements. In writing that book, Bocheński took on not the role of a believer but the role of a logician. He often recalled the prologue of the Gospel of John: "In the beginning was the Word." Word is *logos* in Greek, which also means sense, reason, and logic, which, according to St John, were of God; therefore, they have God's sanction.

Bocheński studied the problems of religion without any fear of going beyond the bounds of science, as in his research he used modern logic, which embodies the ideal of exactness. Seeing that the world is built logically, the language of logic reflects it best. Hence, philosophical and religious arguments ought to be translated into the language of formal logic. It is then that we can see the validity of the reasoning involved and that we can better understand reality.

Bocheński stresses that there is no proof for the truths of faith; moreover, faith does not need any proof. But we have to know what we believe in. Thus, there is a need for analysis, and, therefore, faith needs reason – that is, logic. Sermons that provide directions on how to live in order to achieve salvation are one thing, analysis of our faith is another. Bocheński devoted a great deal of time to proving the usefulness of modern logic for the humanities. Father Bocheński was convinced, just like Łukasiewicz and Tarski, that, after the work of Bertrand Russell,

we cannot do science as we used to. Philosophy and theology should change as well. Traditional theological problems should be reapproached with the use of new scientific methods. According to Bocheński, that shift did, as a matter of fact, lead to a series of discoveries, for instance Russell's analyses concerning analogy or Tarski's definition of truth. This path was followed by several Polish thinkers before the war, namely by Salamucha, Łukasiewicz, Drewnowski, Sobociński, and Bocheński. They believed that the humanities could not develop independently of modern logic, that no science is above logic. They formed a group of Catholic thinkers within the Lvov-Warsaw School, called the Cracow Circle, and believed that Russell's critical attitude towards religion did not discredit his achievements in the field of logic.

The pro-scientific and pro-logical attitude of the Church was evidenced by Father Bocheński by reference to the works of Albertus Magnus or Girolamo Savonarola, but most often to St Thomas Aquinas, whom he considered to be an example for Catholic intellectuals. St Thomas is not important merely because of what he said but also because of how he said it. The *Summa theologiae* is an example of harmony between faith and science. His works have a solid, logical structure.

Father Bocheński began to propagate the use of logic in theology even before the war, with the publication of *Tradycja myśli katolickiej a ścisłość* [The Tradition of Catholic Thought and Precision]. His last public appearance was a lecture entitled *O współczesnym stanie i zadaniach teologii* [On the Current Status and Aims of Theology], delivered at the Warsaw Theology Academy in 1990, as part of his honorary doctorate award ceremony – a lecture that was later published by the academy.

Bocheński claimed that the new criteria of precision discovered in the 20th century needed to be applied to traditional theological questions. I propose to start with an analysis of classic works, for example, those by St Thomas. The correctness of the premises and the correctness of the reasoning should be examined separately. In the paper *O prostocie Boga* [On the Simplicity of God], Bocheński deals with Aquinas's way of reasoning. This article demonstrates that if we accept St Thomas's premises, his proof of the simplicity of God is valid and the thesis is sound.

As Father Bocheński said, God gave us reason, and, in the 20th century, he gave us the methods of formal logic, both of which were given to us to be used.

Thus, let us not be afraid to analyze the Bible and the catechism from the perspective of logic. Logic is not only for logicians – it is for everyone, just as typewriters, the English language, or computers are for everyone. After Bocheński's death, his life's work, *Logiczne studia na Summą teologiczną św. Tomasza* [Logical Analyses of St Thomas Aquinas's *Summa theologiae*], was published in German in 2003. It was the last book published by a representative of the Lvov-Warsaw School belonging to the first generation of Twardowski's students.

Regarding any thesis, the Catholic intellectual should ask two questions: what it means and why. Our faith has to be clear; it cannot be defended with a secret; it has to be defended with the use of logic. In his innovative book *The Logic of Religion*, Father Bocheński claims that if I believe, then I have to understand what I believe in, and I need to be able to communicate and explain my faith to others. A religious person cannot turn away from reason. For a Christian, the world is built logically; it is not chaotic. The world is like an encrypted text. By studying the world, science laboriously discovers and decodes it. That is why one of Father Bocheński's famous sayings is that beyond logic there is only nonsense. Bocheński combined his appreciation for logic with the conviction that, apart from the real world, there is also an ideal one.

Religious people often have certain complexes when discussing logic and faith. These complexes are unjustified. Believers should know that faith contains revealed elements (meaning that a Catholic considers them true because God has revealed them). It is worth mentioning here that, first of all, faith is not a science and it does not need to pretend to be one. Secondly, every science adopts certain concepts without proof, the so-called primitive notions. One might say that for a Catholic sentences given to us through revelation are such primitive concepts. In addition, in faith there are sentences that communicate dogmas, and they act as axioms, as in geometry, for example. We have accepted these dogmas because we believe those who received them to be prophets and saints, whom we consider authorities. Of course, dogmas are accepted only by believers. Theological conceptions, such as Thomism and Scotism, may also be considered axioms. Thus, in theology, in reflection on God, instead of the observation sentences that are found in the natural sciences, we have dogmas formulated by the Church.

According to Bocheński, the formal structure of scientific and religious thinking is similar. The theologian explains the meaning of dogma and organizes it with the use of theological concepts. Instead of observation sentences, he oper-

ates with truths of faith. The sentences that speak of our faith should not be examined with the senses, as in a biology or chemistry lesson, because religion concerns transcendental reality. On the other hand, the sense of religious sentences, their meaning and the consequences that result from them, should be studied. Coming from certain axioms, through reasoning we reach new claims; this is done, for example, by St Paul in his letter to the Corinthians when he uses the resurrection of Christ to prove the resurrection of the dead at the time of the Last Judgement. Thus, it is possible to simultaneously use logic and pose metaphysical and religious questions. The Christian worldview is not scientifically justified because it does not have to be. It contains a synthesis of reality, its evaluation, and answers to existential questions. It is adopted through an act of will. According to Bocheński, for a Catholic, that worldview is justified by a conviction that it orders our world, that without faith the world would be absurd and life would have no meaning.

Father Bocheński enjoyed quoting St Catherine who, as a mystic, experienced God and received a clue from him: think about me, Catherine, and I will think about you. Pray on your knees but do not think about me on your knees! One should pray to God and think about him, but one should think about God properly – following the best methods of logic. This, Bocheński claims, is the attitude of a true Catholic. That is how Christian thought avoids nonsense, unfounded sentences, and heretical statements.

The Position of John Paul II

It was very satisfying for Bocheński when, 10 years after the publication of his book *Między logiką a wiarą* [Between Logic and Faith], Pope John Paul II raised this subject in his encyclical from 1998. It is worth considering how and why there was a change in the attitude of the Church towards science and scientists. It is known that John Paul II met with scholars and that he nullified the document condemning Copernicus's theory. The Pope stated time and again that there is no conflict between faith and science; what is more, this Pope did not want the two to exist apart as independent fields. He postulated treating faith and science as different, complementary ways of pursuing truth. In the first sentence of the encyclical *Fides et ratio* from 1998, he writes: "Faith and reason are like two wings

on which the human spirit rises to the contemplation of truth."[2] In a word, today a scholar like Copernicus would not have a problem with the Pope.

Now, it is worth asking how the Church treats the relationship between faith and science today, with science understood as a method of seeking truth. It is hard not to remember that during his pilgrimage around Poland, on 8 June 1997, John Paul II, when addressing scholars gathered in Kraków, revealed that during his studies he read and drew upon the works of, among others, Father Professor Jan Salamucha. In fact, Father Salamucha is a somewhat forgotten figure, even though he made important contributions to logic and Christian thought. The figure and the work of Salamucha were evoked by John Paul II not without reason. The reference to Salamucha was very significant, as John Paul II set this philosopher as a role model during his long pontificate. It turned out that he was particularly important to the Holy Father, for it was Salamucha who, before the war, along with Bocheński, Łukasiewicz, Sobociński, Drewnowski, and Konstanty Michalski, formed the Catholic part of the Lvov-Warsaw School called the Cracow Circle. They posed questions important to the Pope and conducted studies in a manner that was exemplary in the eyes of the Holy Father.

As I have mentioned, in the encyclical we repeatedly come across the thesis that there is no conflict between faith and reason. If a believer sees a contradiction here, then he is mistaken, because both faith and reason come from God, writes John Paul II. There is no competition between them; they are two different areas of reflection. The encyclical (para. 4) mentions that in philosophy there are certain permanently present principles, that there is a set of philosophical truths, for example, the principles of noncontradiction, purposefulness, and causality. In my opinion this is a recognition of the rules and categories of thinking, and so of the achievements of logic. The Pope again refers to this view (para. 75) when he reminds us that theology needs criteria of rationality and precision, that that is the guarantor of its results. The encyclical contains not only a general recommendation expressed, for instance, in the approval of the opinion of St Augustine, who wrote that faith does not exist without thinking; there is also practical, concrete advice in the encyclical. The role of logic is noted. And it could not be otherwise. If the Pope sees and recognizes the role of reason and thus the scien-

[2] John Paul II, *Encyclical Letter "Fides et ratio" of the Supreme Pontiff John Paul II to the Bishops of the Catholic Church on the Relationship between Faith and Reason*, URL: https://www.vatican.va/content/john-paul-ii/en/encyclicals/documents/hf_jp-ii_enc_14091998_fides-et-ratio.html.

tific method in theology and philosophy, that must signal the adoption of the best tools available to thought, that is to say, logic. Let us remember that in Latin *ratio* has two meanings: reason, usually associated with science, and calculation, that is, calculus. And the so-called propositional calculus in logic is the codification of possible deductive reasonings. Therefore, the title of the encyclical could be interpreted as faith and the classification of reasonings, forms of justification.

The Holy Father brought up Salamucha not merely as a sentimental recollection of his youth. It was how the Pope told us how to work, philosophize, and how to be a modern humanist. The answer is short: like Salamucha and other like-minded members of the Lvov-Warsaw School. According to the Pope, the philosophers of this school may be considered models for the modern humanities. The members of the Cracow Circle ignored the division of universities into disciplines and institutes; they read Aristotle and contemporary thinkers, they knew Greek and Latin as well as French and English, and they were solving classical philosophical questions formulated in antiquity using formal logic created in the 20th century.

Conclusion

Using the example of Father Bocheński's writings and one problem, that is, the relationship between faith and reason, I have tried to present what the thinking of the Catholic philosophers of the Lvov-Warsaw School consisted in.

As follows from the analysis carried out by Bocheński, most of the sentences that we consider true in our scientific and everyday lives are not verified by us personally or sensorily. Most often, we adopt them by relying on experts in a given field. For instance, I trust my doctor, and so I consider what he says about my illness to be true. I think so because the doctor and his teachers have studied my illness. However, it cannot be forgotten that in life we encounter other ways of recognizing sentences as true – for example, a child trusts what their mother says and a lover accepts as true what he hears from his beloved. Here, trust does not result from the fact that someone is an expert and follows scientific procedures, but that they are reliable, becoming authoritative figures in our eyes. For example, believers recognize apostles, mystics, and prophets as having authority, and therefore they consider their testimonies valid. On the other hand, verification

of the sentences that are the object of our faith is logical in character, namely we prove that they do not violate the laws of logic.

Regardless of the philosophical considerations of the relationship between science and faith, we know that this is a problem that continues to evoke emotions in everyday life. In 2014, this issue was the subject of public debate in Poland with respect to the conduct of doctors who did not want to perform abortions. It suffices to trace how the content of the oath taken by doctors has changed. We know what the oath was before the war; it was different in the Polish People's Republic, and today it again has a different content. And there are countries where such an oath no longer exists. This is not the result of medical discoveries but a symptom of changes in thinking about man – changes that have occurred before our eyes. One may wonder whether this new outlook on man and life is really justified and sound. Many people probably do not remember that when the Nuremberg trials began in 1946, the first hearing did not concern Nazi dignitaries but doctors who experimented on people and helped kill prisoners. The conduct of those doctors was then considered so contrary to the norms of European civilization that their trials were scheduled first. I am not sure whether today those matters would be thought of as crimes.

There are currently ongoing attempts to redefine life, its beginning and end, in a very different way than it was defined 50 years ago. The laws regarding abortion and euthanasia adopted in many countries are very different, which proves that those changes have taken place under the influence of ideology, not science. It seems that in debates on these fundamental questions there should also be room for the voices and reflections of philosophers who understand the identity of our civilization and ethical issues better than others. The belief that many professions should be governed by something more than market laws and procedures can hardly be considered outmoded. For several generations it was thought that a good artist, scientist, or doctor is someone who can not only follow the procedures specific to their profession, that in these professions it is not enough to be a skilled tradesman, because when working in these areas we also realize a certain system of values.

The figure of a monk is usually associated with someone boring, isolated from life, who spends all his days in a church or library, reading old, dusty books. The figure of Father Bocheński completely contradicts such stereotypes. He was very active throughout his life. He fought in the war against the Bolsheviks in

1920, he enjoyed life as a student, and as a monk he passionately smoked cigarettes, travelled a lot, and always drove fast. When the war was approaching, he wrote a manual for soldiers entitled *De virtuti militari*. In September 1939, instead of sitting quietly in a monastery, he joined the army and fought under General Franciszek Kleeberg. After the invasion of Poland, using his documents as a professor in Rome, he left Poland for Italy. Then he joined the Polish Armed forces in the West. First, he was in Scotland, then he fought in the Polish Second Corps of General Władysław Anders. He received the rank of lieutenant colonel. After the war, following the will of his monastic superiors, he remained in Western Europe. In addition to his philosophical works, he published a lot on Sovietology. He was considered an eminent expert on Marxism-Leninism in the West and the founder of the Fribourg School of Sovietology. Due to these Sovietological achievements, in the Polish People's Republic it was forbidden to print Bocheński's works or even to quote him. The fact that six countries employed him as an advisor on the fight against communism is evidence of the international recognition he received. Thus, he undertook research in several fields. He never gave up his little joys in life. For example, at the age of 70 he obtained a pilot's license in order to fly himself to lectures across Europe, which he continued to give even after he was 90 years old.

I have tried to show that Father Bocheński and his thought are evidence that philosophy poses interesting questions, that one can be a monk and live with passion, that a Catholic philosopher can lead an interesting and active life, that by serving God with one's intellect, one may remain close to the most important matters of this world, and that we need philosophy.

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ISSN 0860-3839
DOI: 10.14394/edufil.2020.0014
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I.M. Bocheński and Theophrastus' Modal Logic

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Father Innocenty Maria Bocheński[1] expounded his interpretation of Theophrastus' logic in three books: *La logique de Théophraste* (1947), *Ancient Formal Logic* (1951) and *Formale Logik* (1956). According to Bocheński, Theophrastus is a logician who developed and systematized Aristotle's late logical system and departed from his master only while discussing modal syllogistic. Bocheński's general assessment of Theophrastus presupposes a developmental interpretation of Aristotle's logic and a rephrasing of categorical syllogistic in the language of first-order predicate calculus, whereby the Aristotelian letters are interpreted as predicate letters of variables quantified over with the usual Fregean quantifiers.[2] In the same years, Jan Łukasiewicz was interpreting Aristotle's logic in an analogous way.[3] Our understanding of Aristotle's logic is now different from Bocheński's,

[1] For the sake of consistency, I will be referring to the great Polish historian of logic by his religious name as a Dominican ("Innocenty") even though some of his writings were published under his baptismal name ("Józef").

[2] See, e.g., Bocheński's interpretation of prosleptic propositions in I.M. Bocheński, *La logique de Théophraste*, Fribourg 1947, p. 49 : "Théophraste enseignait aussi – toujours d'après Alexandre – que les formules avec et sans quantificateur, c'est-à-dire les formules
C φxψx
C Πxφx Πxψx
sont équivalentes."
In the current logical notation, the two formulae are respectively: (i) $\varphi x \rightarrow \psi x$ and (ii) $\forall x\, \varphi x \rightarrow \forall x\, \psi x$.

[3] See especially J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd ed., Oxford 1957. Even though he has been accused by contemporary historians of logic of making historically inaccurate statements, Łukasiewicz was very careful in his wording. He

and many scholars favour a "mereological" interpretation of Aristotle's syllogistic.[4] This has an obvious implication for our assessment of Bocheński's overall interpretation of Theophrastus: if we choose a mereological interpretation of Aristotle's syllogistic, Theophrastus' "extensional" project becomes a radical novelty in the history of logic.[5] But this characterization of the logic of the philosopher of Eresus is probably as inaccurate as the extensional reading of Aristotle. Leaving aside Bocheński's overall evaluation of Theophrastus' contributions to logic, we can still appreciate the ambitious programme of the Greek philosopher, if we pay attention to the *minutiae* of the reading offered by the Polish Dominican, even though *La logique de Théophraste* was published more than 70 years ago.

After Bocheński's groundbreaking works, there have been many publications on Theophrastus' logic.[6] An incomplete list includes:

1) a collection of the logical fragments by Luciana Repici;[7]

maintained that Aristotle introduced "variables" by employing letters (cf. p. 7) but never claimed that Aristotle's letters are quantified over. On the contrary, Łukasiewicz wrote: "Aristotle had no clear idea of quantifiers and did not use them in his works; consequently we cannot introduce them into his syllogistic" (p. 83). However, he went on to say that "there are two points in his [*sc.* Aristotle's] system which we can understand better if we explain them by employing quantifiers" (pp. 83–84). For this reason, Łukasiewicz maintained that Aristotle's syllogistic applies only to non-empty terms (for a criticism of this latter claim, see S. Read, *Aristotle and Łukasiewicz on Existential Import*, "Journal of the American Philosophical Association" 2015, Vol. 1, No. 3, pp. 535–544). Bocheński maintains that Aristotle's analysis of universal propositions in *Prior Analytics* A, 41, 49b14–20 is similar to a Frege-style formalization: "Hier wird eine Analyse der Aussage 'A kommt jedem B zu' vorausgesetzt, die folgendermaßen gedeutet werden könnte: 'Für jedes x: wenn B dem x zukommt, dann kommt A dem x zu'; es würde sich dann um die modern formale Implikation handeln" (I.M. Bocheński, *Formale Logik*, Fribourg–München 1956, p. 92).

4 See especially M. Malink, *Aristotle's Modal Syllogistic*, Cambridge, MA 2013. Against Malink's interpretation, see J. Barnes, *Truth, etc.*, Oxford 2007, pp. 386–419.

5 I have some reservations on this interpretation, as will be clear in what follows.

6 *La logique de Théophraste* includes an excellent survey of the sources for the reconstruction of Theophrastus' logic (cf. pp. 15–38). Bocheński was aware that a new collection of his logical fragments was a scholarly need: "La seule collection presque complete des fragments de la logique de Théophraste se trouve dans *Prantl*. Fr. Wimmer, *Theophrasti Eresii Opera quae supersunt omnia, t. III, fragmenta continens*, Lipsiae 1872 ne donne qu'une très petite partie de l'ensemble: tandis que *Prantl* cite plus de 100 fragments logiques, Wimmer n'en a que 16. *Prantl* attribue souvent à Théophraste des textes manifestement stoïciens. Une collection nouvelle serait désirable" (p. 15, n. 19; "Prantl" obviously refers to C. Prantl, *Geschichte der Logik im Abendlande*, Leipzig 1927).

7 L. Repici, *La logica di Teofrasto. Studio critico e raccolta dei frammenti e delle testimonianze*, Bologna 1977. Fortenbaugh's collection of fragments (see next footnote) does not entirely supersede Repici's collection (cf. on this L. Gili, *La sillogistica di Alessandro di Afrodisia. Sillogistica categorica e sillogistica modale nel commento agli* Analitici Primi *di Aristotele*, Hildesheim 2011, p. 35).

2) a complete collection of all extant fragments of Theophrastus' oeuvre by William W. Fortenbaugh and other scholars;[8]
3) a series of seminal studies by Mario Mignucci on Theophrastus' logic;[9]
4) Pamela Huby's commentary on fragments 68–136 Fortenbaugh (on logic).[10]

As Katerina Ierodiakonou makes clear in her recent entry on Theophrastus for the *Stanford Encyclopedia of Philosophy*,[11] these studies have largely changed our understanding of Theophrastus' logic since the publication of *La logique de Théophraste* in 1947. But Bocheński's contributions can still open new paths to the historical research on Theophrastus' logic precisely because our understanding of Aristotle's modal syllogistic has also changed in recent years. In what follows, I will argue that Bocheński's reading presents Theophrastus as a much more revolutionary logician than the Polish Dominican was able to acknowledge. I will focus in particular on Theophrastus' modal logic, and I will show that Theophrastus laid the foundations for a different syllogistic: in my opinion, he did not abandon the "mereological approach" altogether (*pace* Bocheński's extensional reading), but proposed a logic that is conceived as a deductive system where the deducibility of an argument rests merely on the syntax of the language, whereas the Aristotelian validity seems to involve also semantic considerations, as is clear in the case of the proof of validity of the syllogistic mood *Barbara LX-L*. Such a system is not primarily designed as a theory of deduction for any of the Aristotelian sciences. Owing to this proposal, Theophrastus was probably one

8 W.W. Fortenbaugh et al., eds., *Theophrastus of Eresus: Sources for His Life, Writings, Thought and Influence*, 2 vols., Leiden 1992.

9 M. Mignucci, *Per una nuova interpretazione della logica modale di Teofrasto*, "Vichiana" 1965, Vol. 2, pp. 3–53; M. Mignucci, *Theophrastus' Logic*, in: *Theophrastus: Reappraising the Sources*, eds. J. van Ophuijsen, M. van Raalte, Leiden 1998, pp. 39–65.

10 P. Huby, *Theophrastus of Eresus: Sources for His Life, Writings, Thought and Influence. Commentary Volume 2. Logic*, with contributions on the Arabic material by D. Gutas, Leiden 2007.

11 Cf. K. Ierodiakonou, *Theophrastus*, in: *The Stanford Encyclopedia of Philosophy*, ed. E. Zalta, URL: https://plato.stanford.edu/entries/theophrastus (substantive revision published on 24.09.2020): "There has been a scholarly debate about whether Theophrastus recognized arguments in *modus ponens*, *modus tollens*, *modus ponendo tollens* and *modus tollendo ponens*:
If *p*, then *q*; but *p*; therefore *q*.
If *p*, then *q*; but not-*q*; therefore not-*p*.
Either *p* or *q*; but *p*; therefore not-*q*.
Either *p* or *q*; but not-*q*; therefore *p*.
While earlier commentators doubted that Theophrastus ever considered anything of the sort (see Bocheński 1947), recent scholars have maintained that he studied such arguments, or at least that he studied arguments which can be regarded as their forerunners."

of the main sources of inspiration for the 1st-century BC Aristotelian Boethus of Sidon.[12] Bocheński's adoption of the language of first-order logic to formalize Theophrastus' claims might now be called into question, but thanks to this "old-fashioned" methodology we can better appreciate the originality of the contribution of the philosopher of Eresus to the historical development of syllogistic.

Bocheński on Theophrastus' Overall Contribution to Logic

Bocheński's interest in Theophrastus' logic goes back to the 1930s.[13] In 1937, he published an article on Aristotle's and Theophrastus' understanding of modal propositions.[14] In 1939, he was about to publish a monograph in French on Theophrastus' logic in the series Collectanea Logica edited by Łukasiewicz (vol. 1, pp. 195–304, with the title *La logique de Théophraste*). The typescript and the print version were both destroyed during the German bombing of Warsaw in September 1939. The only surviving copy sent to Heinrich Scholz was later destroyed during the Allied bombing of Münster in 1943. In 1944 Bocheński found in Rome incomplete proofs of his book and started working on it again. *La logique de Théophraste* would finally appear in 1947 (Librairie de l'Université, Fribourg en Suisse).[15] Bocheński wrote again on Theophrastus in 1951 in *Ancient Formal Logic* and in 1956 in his famous *Formale Logik*. In these two latter books, Father Bocheński devoted only a few pages to Theophrastus, but these sketchy remarks are useful to reconstruct what he maintained to be the core contribution to logic of the philosopher of Eresus. In his 1951 book, Bocheński wrote:

> In the light of the preserved fragments, we see that the work of Theophrastus consisted mainly in the development of the doctrines of Aristotle in the manner of Aristotle's own late writings. By doing so, Theophrastus contributed considerably to the formation of what was later called "classical logic" and

[12] I argued in favour of this claim in L. Gili, *Boeto di Sidone e Alessandro di Afrodisia intorno alla sillogistica aristotelica*, "Rheinisches Museum für Philologie" 2011, Vol. 154, pp. 375–397.

[13] On Bocheński's scholarly activity in Rome at the Pontifical University of St. Thomas Aquinas (the "Angelicum"), see E. Kaczyński, *La ricerca logica di I.M. Bocheński durante il suo insegnamento all' "Angelicum" (1934–1939)*, "Angelicum" 2003, Vol. 80, No. 1, pp. 9–33.

[14] I.M. Bocheński, *Notes historiques sur les propositions modales*, "Revue des sciences philosophiques et théologiques" 1937, Vol. 26, No. 4, pp. 673–692.

[15] I take this information from I.M. Bocheński, *La logique de Théophraste*, op. cit., pp. 5–6.

> perhaps also opened the path to the Stoic-Megaric Logic. At the same time, however, it must be stressed that his teaching contains several un-Aristotelian elements, especially in modal logic.[16]

In the short notes of the *Formale Logik*, the Polish Dominican makes analogous claims.[17] With the expression "classical logic," Bocheński does not refer to first-order predicate calculus, but to the traditional "Aristotelian logic" that was taught across Europe since the discovery of the *logica nova*. According to Bocheński, Theophrastus followed in the footsteps of his master and systematized his system, thereby making it suitable for handbook expositions. Theophrastus developed new ideas only in modal logic. Bocheński argues in detail for this interpretation in his 1947 book as well. At this level of generality, one could hardly say that Bocheński's picture needs any revision. It is worth noting, however, that the Polish Dominican took for granted that both Aristotle's and Theophrastus' treatment of quantified propositions could be translated into the language of lower predicate calculus, where a sentence like "All Dominicans pray the rosary" becomes a material implication of this form:

(1) $\forall x\ (Ax \rightarrow Bx)$ (where the predicate letters "A" and "B" stand respectively for "Dominican" and "is praying the rosary").

Far from being a mere logical tool to analyze the sentences, the formalization in the language of first-order logic had profound philosophical implications. In Bocheński's view, both Aristotle and Theophrastus were working with a paradigm where individual variables are quantified over, predicate letters are not, and universal affirmative propositions involve a material implication. Aristotle and Theophrastus were obviously not aware of the doctrine of quantification, but they worked with this scheme in mind. Bocheński saw a deep philosophical reason for this reading:

> It is worth noting that after Plato the logical analysis of propositions reaches a third degree of subtlety with κατὰ πρόσληψιν propositions: the young Aris-

[16] I.M. Bocheński, *Ancient Formal Logic*, Dordrecht 1951, p. 72.

[17] Cf. I.M. Bocheński, *Formale Logik*, op. cit., p. 114: "Er [*sc.* Theophrastus] hat, erstens, verschiedene Lehren seines Meisters so entwickelt, daß er sozusagen die spätere 'klassische' Logik vorbereit hat; der aristotelischen modalen Syllogistik hat er, zweitens, seine eigene ganz andere gegenübergestellt; endlich entwarf er eine Lehre vom hypothetischen Syllogismus, welche die megarisch-stoische Lehre vorbereitete."

> totle identifies only two elements, i.e. the subject and the predicate (the copula does not play any significant role); in the *Analytics*, the copula is already one of the three necessary elements of the proposition; finally, in our author [*sc.* Theophrastus], we find four elements: two terms, that correspond to the subject and the predicate, an undetermined substrate and the [material] implication that replaces the copula. […] We should add an extra-logical remark about the origin of our propositions [i.e. κατὰ πρόσληψιν propositions]. They seem to fit Aristotle's metaphysics, because we know that Aristotle was distinguishing two elements in every empirical object, i.e. an undetermined substrate or matter and a form that determines this substrate. Our propositions seem to be a transposition of this doctrine in the logical space, because they also divide the subject and the predicate into two elements, i.e. an undetermined x, i.e. the substrate, and its determination.[18]

I am inclined to argue that Bocheński was not able to appreciate in full the philosophical meaning of Theophrastus' contribution to logic precisely because of the mistaken assumption according to which predicate letters and variables

[18] I.M. Bocheński, *La logique de Théophraste*, op. cit., pp. 50–51 : "Il est intéressant de constater qu'avec les propositions κατὰ πρόσληψιν c'est un troisième degré de subtilité que l'analyse logique de la proposition atteint depuis Platon : chez le jeune Aristote, on y distingue deux éléments seulement, un sujet et un prédicat – la copule ne joue aucun rôle important ; dans les Analytiques, elle est déjà un des trois facteurs nécessaires de la proposition ; enfin chez notre auteur [*sc.* chez Théophraste] nous en trouvons quatre : deux termes, qui tiennent la place du sujet et du prédicat, le substrat indéterminé et l'implication substituée à la copule. […] Une remarque extralogique s'impose quant à l'origine de nos propositions. Elles semblent notamment bien correspondre à la métaphysique aristotélicienne : on sait, en effet, qu'Aristote distinguait en tout objet empirique deux éléments, un substrat indéterminé, la matière, et une forme qui détermine ce substrat. Nos propositions semblent une transposition de cette doctrine dans l'ordre logique, car elles divisent aussi le sujet et le prédicat en deux éléments, un x indéterminé qui est substrat et une détermination" (my translation). Prosleptic propositions are propositions where a third term is (implicitly) introduced. The prosleptic counterpart of "*B* is said of all *A*" is "*B* is said of all of which *A* is said" or "For every *X*, if *A* is said of all *X*, then *B* is said of all *X*." Aristotle introduced prosleptic propositions in *Pr. An.* B, 5, 58a29–30, but did not use the expression κατὰ πρόσληψιν that appears to have been introduced by Theophrastus (cf. Alex. Aphr. *In An. pr.* 378.14; for an overview of the ancient sources on prosleptic propositions and prosleptic syllogisms, cf. W. Kneale, M. Kneale, *Prosleptic Propositions and Arguments*, in: *Islamic Philosophy and the Classical Tradition: Essays Presented by His Friends and Pupils to Richard Walzer on His Seventieth Birthday*, eds. S.M. Stern, A. Hourani, V. Brown, Columbia, SC 1972, pp. 189–207). According to Malink, Aristotle might have been aware of this classification (cf. M. Malink, *Figures of Prosleptic Syllogisms in* Prior Analytics *2.7*, "Classical Quarterly" 2012, Vol. 62, No. 1, pp. 163–178).

belong to different semantic types. I think that neither Aristotle nor Theophrastus would have accepted a reading of quantified propositions where

> x is a zero-order individual variable, and A and B are first-order predicates. In the standard first-order models, the semantic value of zero-order terms is an individual, and the semantic value of first-order predicates is a set of individuals.[19]

Both Aristotle and Theophrastus had a mereological understanding of letters in their syllogistic, as Marko Malink has argued at length in his publications.[20] According to this reading, all the letters appearing in the *dictum de omni et de nullo* stand for parts and not for individuals. Since the validity of syllogisms rests on the *dictum de omni et de nullo*, we could not say that the semantic value of a zero-order term in any premise or conclusion of a valid mood is an individual, while the semantic value of a first-order predicate is a part. But Bocheński was right about a crucial point: there is a historical direction towards a type of logic that is freer from its purpose of serving as the deductive system of a particular science. A logic that is not designed for an Aristotelian science could eventually be treating individual variables as belonging to a lower semantic type than predicate letters. *Pace* Bocheński, Theophrastus did not make this last step, but certainly laid the foundations for this possible outcome in the historical development of logic. Interestingly, Theophrastus was in all likelihood the main source of inspiration for Boethus of Sidon, who might have been thinking of prosleptic propositions when he argued *contra Aristotelem* that the syllogisms in the three figures are all complete.[21] Boethus still advances a heterodox reading of the *dictum de omni et de nullo*, that is, a reading whereby all letters and variables have the same semantic value (a universal), but his philosophy seems to be in need of quantifiers for individuals.[22]

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[19] M. Malink, *A Non-Extensional Notion of Conversion in the* Organon, "Oxford Studies in Ancient Philosophy" 2009, Vol. 37, p. 110.

[20] See again M. Malink, *Aristotle's Modal Syllogistic*, op. cit.

[21] I take the liberty to refer again to L. Gili, *Boeto di Sidone e Alessandro di Afrodisia*, op. cit.

[22] Marwan Rashed makes this claim in his paper on Boethus' syllogistic (pp. 255–289) in R. Chiaradonna, M. Rashed, *Boéthos de Sidon – Exégète d'Aristote et philosophe*, Commentaria in Aristotelem Graeca et Byzantina – Series Academica 1, Berlin–Boston, MA 2020: "(*dictum* hétérodoxe) *A est prédiqué* κατὰ παντός *de B* ssi *il n'y a pas de partie de B dont A ne soit pas prédiqué* Boéthos, qui affirme, dans son ontologie, la préséance de l'individu sur ses déterminations formelles, semblerait n'avoir aucune raison de privilégier une lecture de ce type" (p. 283).

If we re-read Bocheński's book with these historical insights in mind, we cannot but appreciate how the Polish Dominican anticipated many of these conclusions in his reconstruction of Theophrastus' modal logic.

Bocheński on Theophrastus' Modal Logic

Even though I am not suggesting that Theophrastus endorsed an extensional reading of propositions, there are certainly hints that the philosopher of Eresus might have been open to a logic that is free from its function to serve as the underlying deductive system of one of the Aristotelian sciences. In so doing, Theophrastus was probably not looking at the predicative relations between subjects and predicates as the grounds for the modality of a premise or a conclusion, but was offering a mere syntactic characterization of modalities. This is not tantamount to stating that Theophrastus abandoned the mereological approach of his master, but he certainly did abandon the *rationale* for a mereological approach, that is, the idea that premises and conclusions always express a predicative relation between two *praedicabilia*. The *praedicabilia* always denote concepts, that is, classes. If all the terms appearing in a syllogism are predicables, they all refer to classes, not to individuals.

There are also hints that Theophrastus might have favoured a merely extensional reading of propositions. The best example is Theophrastus' proof for the validity of the law of conversion for universal negative propositions. Aristotle had to introduce ecthesis to prove this law – a procedure that is certainly problematic for readers who translate Aristotle's sentences into the language of first-order predicate calculus.[23] Theophrastus claimed to have a simpler proof for the conver-

[23] Simply put, either Aristotle has to use the law of conversion for particular affirmative propositions to demonstrate by ecthesis the law of conversion of universal negative propositions (but this would be a circular reasoning, inasmuch as the law for particular affirmative propositions is demonstrated by means of the law of conversion for universal negative propositions) or he has to use extralogical notions (cf., e.g., J. Łukasiewicz, *Aristotle's Syllogistic*, op. cit., p. 60: "[A] proof by perception is not a logical proof"). Łukasiewicz and Bocheński take the ecthetic conversion of particular affirmative propositions to be self-evident, thereby adopting a strategy similar to the one of Theophrastus (who took as self-evident the law of conversion for universal negative propositions): "Chez Aristote, la preuve est 'ecthétique' et peut être résumée ainsi: La thèse à prouver est 'EYbaYab', suppose qu'elle soit fausse, 'Yba' implique 'NYba'; or, ceci implique à son tour qu'il existe (au moins) un *c* tel, qu'il est à la fois un (élément de la classe) *a* et un (élément

sion of universal negative propositions.[24] Bocheński does not develop any extensional reading of this proof probably because that was rather obvious within the framework of his reading of Theophrastus. But it is certainly worth noting that, for Theophrastus, "No *A* is *B*" means that *A* and *B* are "separated" – and one can hardly avoid thinking of non-overlapping diagrams representing the extensions of *A* and *B*. It is still possible that the diagrams would have included the "parts" of the notions *A* and *B*, but I daresay that we are more naturally inclined to think that they would have represented classes of individuals.

In this case, Bocheński's extensional reading, although controversial, is certainly enlightening. The picture becomes more complex if we look at modal syllogistic. In his chapter on Theophrastus' modal logic (pp. 67–102), Bocheński begins by summarizing Aristotle's modal theses. In Bocheński's reconstruction, Theophrastus introduced two novelties in his system:[25]

1) he replaced two-sided possibility with one-sided possibility;
2) he introduced the "*peiorem* rule" (*peiorem semper sequitur conclusio partem*) in virtue of which moods such as *Barbara LX-L* are taken to be invalid, *pace* Aristotle, *Prior Analytics* A, 9 (where such moods are taken to be valid). *Barbara LX-L* is a mood where the major premise is a necessary universal affirmative proposition, the minor premise is a categorical uni-

de la classe) *b*; et s'il en est ainsi, il n'est pas vrai qu'aucun *b* n'est *a*" (I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 55; cf. also J. Łukasiewicz, *Aristotle's Syllogistic*, op. cit., p. 61).

[24] Both Alexander and Philoponus present this proof by Theophrastus (cf. I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 54, n. 201). For a comparative analysis of these two sources I take the liberty to refer the reader to L. Gili, *Il confronto di Giovanni Filopono con Alessandro di Afrodisia intorno al problema della conversione delle proposizioni*, "Elenchos. Rivista di studi sul pensiero antico" 2015, Vol. 36, No. 2, pp. 317–339.

[25] Cf. I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 95 : "Nous avons noté les différences suivantes entre le système de la modalité de Théophraste et celui d'Aristote :

(a) Tandis que chez Aristote '*Mp*' est défini par l'équivalence '*EMpKNSpNSNp*', chez Théophraste il y a presque partout le sens déterminé par la formule '*EMpNSNp*'.
(b) Tandis qu'Aristote admet des modes syllogistiques dans lesquels la conclusion est 'plus forte' qu'une des prémisses (par exemple '*CKSpZqSp*'), chez Théophraste la règle du *peiorem* est rigoureusement appliquée à tous les modes.
(c) La conséquence de (b) et surtout de (a) est que le système de Théophraste est beaucoup plus homogène que celui d'Aristote et contient presque exclusivement des thèses analogues à celles de la logique assertorique; Aristote a, par contre, dans sa logique de la modalité beaucoup de thèses qui n'ont pas d'analogues assertoriques et beaucoup de thèses assertoriques sans analogues dans certains groupes de thèses de la logique de la modalité" (Bocheński's formula "*CKSpZqSp*" in (b) might be a *lapsus calami* for "*CKSpZqSr*," if the Polish Dominican intended to refer to a *Barbara LX-L* mood).

> versal affirmative proposition, and the conclusion is a necessary universal affirmative proposition. This mood is a valid syllogism for Aristotle, but not according to Theophrastus, who maintained that the conclusion must always have the weaker property, which can be found in any of the premises: if one of the premises is negative, the conclusion will also be negative; if one of the premises is particular, the conclusion will be particular; if one of the premises is not necessary, the conclusion will not be necessary.

According to Bocheński, Aristotle conceives of the possibility operator as one-sided possibility chiefly in his *De interpretatione*, whereas in the *Prior Analytics* he mostly uses two-sided possibility.[26] If Theophrastus worked on Aristotle's *last* logical system, we could conclude that the composition of the *Prior Analytics* precedes that of the treatise *De interpretatione*.[27] This historical claim presupposes the questionable idea that (a) Aristotle employs two-sided possibility in the *Prior Analytics* and one-sided possibility in *De interpretatione*, and (b) that Theophrastus wanted to develop the last system proposed by Aristotle. But Bocheński was right in stressing that the philosopher of Eresus does not seem to use two-sided possibility in his modal syllogistic.

A few years before the publication of *La logique de Théophraste*, Albrecht Becker published a landmark contribution to the study of Aristotle's modal syllogistic, *Die aristotelische Theorie der Möglichkeitsschlüsse*.[28] In that book,[29] Becker maintained that Aristotle's *Barbara LX-L* mood is valid because the necessity operator is read *de re*, whereas Theophrastus takes the same mood to be invalid because for him

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[26] Let "M_1" be the operator for one-sided possibility and "M_2" the operator for two-sided possibility. It is possible to define the two operators as follows:
$M_1p \equiv \Diamond p$
$M_2p \equiv \Diamond p \wedge \Diamond \neg p$
On two-sided and one-sided possibilities in *De interpretatione*, see M. Malink, *Aristotle on One-Sided Possibility*, in: *Logical Modalities from Aristotle to Carnap: The Story of Necessity*, eds. M. Cresswell, E. Mares, A. Rini, Cambridge 2016, pp. 29–49.

[27] Cf. I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 102: "On sait que Théophraste suit Aristote dernière manière. Sa logique de la modalité serait donc un argument pour placer le *Per 13* [*sc*. *De interpretatione* 13] après les *APr.*"

[28] A. Becker, *Die aristotelische Theorie der Möglichkeitsschlüsse. Eine logisch-philologische Untersuchung der Kapitel 13–22 von Aristoteles*' Analytica Priora *I*, Berlin 1933.

[29] On Becker's interpretation of Aristotle's modal syllogistic, see L. Gili, *Interpreting Aristotle's Modal Syllogistic*, "Documenti e studi sulla tradizione filosofica medievale" 2015, Vol. 26, pp. 1–12, and L. Gili, *La sillogistica del necessario in alcune interpretazioni novecentesche*, "Rivista di filosofia neoscolastica" 2016, Vol. 2, pp. 445–463.

the necessity operator should be interpreted *de dicto*. The laws of conversions for modal propositions, however, are valid only under a *de dicto* reading of the operators, and Aristotle's modal syllogistic turns out to be inconsistent in Becker's interpretation. Bocheński was not satisfied with the conclusions of Becker's book. According to the Polish Dominican, a *Barbara LX-L* mood is "intuitively correct" – its validity does not hinge on a *de re* reading of the necessity operator. The modal operators are rather defined on the basis of their use in the context of valid syllogisms.[30] *Prima facie*, Bocheński's analysis may seem superficial, but it perfectly fits Malink's suggestion that the minor premise of a *Barbara LX-L* expresses an essential predication, inasmuch as the middle term should either be a definition or a genus or a species. Within this framework, Aristotle's enterprise appears to be consistent and Bocheński is right in stressing that it is only by chance that the validity of *Barbara LX-L* can also be demonstrated if we assume that the necessity operator is *de re* in both the major premise and the conclusion.

Aristotle was guided by the idea that modal propositions express the predicative relationships between *praedicabilia* presumably because he conceived of his modal logic as the underlying deductive system of his theoretical sciences, such as his physics.[31] Bocheński is right in stressing that Theophrastus' system is original and aims at simplicity.[32] In the light of recent contributions on Aristotle,[33] we could hypothesize (a) that Theophrastus no longer grounded his modalities on the different types of predication generated by the combination of the *praedicabilia*. And if this were proven to be the case, one could further argue (b) that Theophrastus wanted to develop a modal system independently of its use in the context of one of the Aristotelian sciences. Future research will have to assess

[30] Cf. I.M. Bocheński, *La logique de Théophraste*, op. cit., pp. 97–98: "Aristote n'aurait pas pensé à une structure déterminée des propositions modales, si non dans quelques passages ajoutées plus tard. Ses thèses sont basées sur une certaine intuition de la possibilité et du syllogisme; il suffit d'y penser un peu et l'on voit immédiatement que A 5–7 aussi bien, par exemple, que le mode '*CKSUmaZUbmSUba*' [i.e. *Barbara LX-L*] paraissent intuitivement correct. C'est par hasard que certaines de ces thèses se déduisent parfaitement de l'hypothèse beckérienne."

[31] Mauro Mariani argued for this claim in *Logica modale e metafisica. Saggi aristotelici*, Pisa 2018, pp. 61–84 ("Semantica aristotelica e sillogistica modale") and pp. 193–215 ("Sillogistica modale e teorie della predicazione").

[32] Cf. I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 94 : "En somme, la syllogistique modale de Théophraste devait être, quant aux thèses, remarquablement plus simple que celle d'Aristote."

[33] I refer mostly to Malink's and Mariani's books (cited respectively in footnotes 4 and 31). For additional references, see L. Gili, *Interpreting Aristotle's Modal Syllogistic*, op. cit.

each of these two claims, but it is worth noting that Bocheński's interpretation undoubtedly provokes these reflections and these working hypotheses.

In conclusion, Theophrastus developed a new modal syllogistic, probably because he had a different understanding of the possibility operator and of syllogistic validity.[34] While Aristotle uses both one-sided and two-sided possibility, Bocheński argues that Theophrastus chooses to employ only one-sided possibility. While addressing syllogistic validity, Bocheński writes that

> in Aristotle's system, the middle term (in the first figure) is qualified by the major term and the minor term is conceived of as an element of the class represented by the middle term, whereas in Theophrastus' system, the three terms are looked at as extensions that are united or "separated" among each other; this union or separation may be either common, necessary or possible.[35]

According to Bocheński, the letters represent the extensions of classes in both Aristotle and Theophrastus. In Aristotle's system, however, the major term is supposed to "qualify" the middle term in the first figure – and this suggests that Aristotle had in mind the real-world application of logic as a tool for capturing real relations. In Theophrastus, on the other hand, this guiding idea appears to be absent: the philosopher of Eresus is merely interested in the relations between the classes represented by the letters.

Conclusion

The extensional reading of Theophrastus' logic is the most natural outcome for a historian of logic who happened to be writing in the 1930s and 40s, when first-order logic was taken to be *the* logic and any argument had to be formalized in the language of *the only logic* available. Bocheński did not only employ the lan-

[34] Cf. I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 98: "Théophraste, en changeant beaucoup de choses dans la logique de la modalité aristotélicienne, s'est probablement laissé guider non pas par une idée de la structure différente, mais par une autre intuition de la possibilité et du syllogisme."

[35] I.M. Bocheński, *La logique de Théophraste*, op. cit., p. 100: "[T]andis que chez Aristote le terme moyen se trouvait (en 1re figure) qualifié du terme majeur et le terme mineur était conçu comme un élément de la classe représentée par le terme moyen – chez Théophraste les trois termes sont considérés en extension, unis ou 'séparés' entre eux; cette union ou séparation peut être commune, nécessaire ou possible" (my translation).

guage of the lower predicate calculus as a tool to formalize Theophrastus' theses, but maintained that the philosopher of Eresus had an extensional understanding of the relations expounded by the propositions occurring in a syllogism. In the light of more recent studies on Aristotle's modal logic, we might be wary of this conclusion. But thanks to his outdated methodology, Bocheński was able to show an undeniable tendency in Theophrastus' oeuvre, that is, the attempt at creating a logical system where syllogistic validity was no longer grounded on the theory of predication. In making this claim, Bocheński was definitely on the right track.

In future studies, scholars will have to establish whether Theophrastus maintained that the three terms that figure in a prosleptic proposition belong to the same semantic type or to different semantic types (as Bocheński believed). But even if Theophrastus followed his master in offering a mereological foundation for syllogistic, as I believe to be the case, there is a striking difference in the approaches of the two philosophers. As Bocheński rightly observed, Aristotle's remarks on the validity of *Barbara LX-L* are intuitively correct, but if we want to make sense of this intuition, we must admit that all the letters that appear in a syllogistic premise and in its prosleptic counterpart belong to the same semantic type and are one of the predicables expounded in the *Topics*. Accordingly, the major premise of *Barbara LX-L* is necessary because it expresses an essential predication or the predication of a *proprium*. If this is the case, the middle term can only be a definition, or a genus, or a difference, or a *proprium*, but such a term can only appear in an essential predication or in the predication of a *proprium* under the hypothesis that a syllogistic premise only deals with *per se* predications.[36] Aristotle can state the validity of *Barbara LX-L* only on the basis of these semantic considerations. As Bocheński remarked, Theophrastus has a simpler system, where a mood is valid only insofar as it is deductible from the basic

[36] It is obviously conceivable to form a syllogism where the minor premise expresses an accidental predication:

(a) Able to laugh is necessarily said of all human beings.
(b) Walking is said of all those who are able to laugh.
Therefore, (c) walking is (necessarily?) said of all human beings.

In this example, the major premise is necessary because it expresses the predication of a *proprium*. The minor premise, however, does not express a *per se* predication, but rather an accidental predication, whereby *A* is said of *B*, because there is a *C* to which both *A* and *B happen* to belong. In this case, one can hardly see how it would be possible to infer a necessary conclusion. Hence, it seems reasonable to stipulate that Aristotle would only consider *per se* predications in his syllogistic.

rules of the system, regardless of any semantic consideration about the types of predication that are expressed in the syllogistic premises and conclusions. The extensional reading proposed by the Polish Dominican helps us understand this simplicity. Furthermore, an extensional reading is at odds with the goal of syllogistic to serve as the underlying deductive theory of an Aristotelian science because Aristotelian sciences are not about individuals, but about genera and species. Did Theophrastus want to develop a logic for its own sake, regardless of its use in a scientific context? I have hinted in this paper that I think this was the case. If this claim were to be demonstrated in future studies on Theophrastus' logic, Bocheński will have to be credited for pointing us in the right direction.[37]

Acknowledgements

I thank Mauro Mariani, Lorenz Demey and the two anonymous reviewers of the journal for their comments on this paper. Any remaining mistakes are only mine.

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[37] It could be objected that any extensional reading of Theophrastus' logical fragments would have brought about the same result, but Bocheński has the historical merit of having been the first scholar who consistently applied this method in order to interpret the texts of the philosopher of Eresus.

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Summary

Innocenty Maria Bocheński expounded his interpretation of Theophrastus' logic chiefly in his book *La logique de Théophraste* (1947). In Bocheński's reconstruction, Theophrastus worked on the last insights of Aristotle's syllogistic and systematized it, thereby opening the door to later (Stoic) developments in the history of logic. A closer look at Bocheński's interpretation of Theophrastus' logic can lead us to reassess the originality of the contribution of the philosopher of Eresus. As more recent studies have convincingly shown, Aristotle's modal system is grounded on the theory of predication expounded in the *Topics*. The validity of *Barbara LX-L* rests on the essential predications that the major premise and the conclusion are descriptive. According to Bocheński, Theophrastus had an extensional understanding of logic, as is clear from his proof for the rules of conversion of categorical universal propositions. Bocheński also stresses that Theophrastus consistently avoids Aristotle's two-sided possibility and this might also be read as an attempt to develop a self-contained logical system that is not merely seen as the deductive system of a theoretical discipline. Bocheński's overall assessment of Theophrastus' logic might be in need of revision, inasmuch as our understanding of Aristotle's logical enterprise has radically changed in the last decades, but the *minutiae* of Bocheński's reading of Theophrastus are compelling and can stimulate new studies on the successor of the Stagirite.

Key words: I.M. Bocheński, Theophrastus, modal syllogistic, *Prior Analytics*, Aristotle, modal logic

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Logical Analysis against Superstitions: Józef M. Bocheński on the Social Role of Philosophy

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1. Introductory Remarks[1]

The writings of Józef M. Bocheński are full of bon mots and slogans. This is one of them:

(1) A philosopher analyzes, not moralizes.[2]

Bocheński was a theorist and a practitioner, as well as an apologist, of philosophical analysis. He was convinced that real progress in philosophy is possible only through the application of analytic methods to small problems.

Bocheński wrote elsewhere:

(2) Philosophy has a demonic task: to destroy […] superstitions.[3]

Here, Bocheński reveals what social role may be played by philosophy in the contemporary world.

Bon mots and slogans, even if they are expressed by the same person, are often difficult to assemble into a coherent vision. This paper's aim is to show how

[1] I would like to thank the Reviewers for their comments that helped me to improve this paper. A shortened, simplified Polish version of this paper was published in the journal *Filozofuj!* (6/2020).

[2] J.M. Bocheński, *Między logiką a wiarą. Rozmowa z Janem Parysem* [Between Logic and Faith: An Interview with Jan Parys], Montricher 1988, p. 98. Unless otherwise stated, all translations from Polish are my own.

[3] Ibid., pp. 85–86.

to combine statement (1) with statement (2), namely to determine in what sense philosophical analysis may serve as a weapon in the battle against superstitions.

In § 2, the concept of analysis, logical analysis in particular, is characterized in the spirit of Bocheński. In § 3, the concept of superstition is defined and commented on. Taking Bocheński's definition as the point of departure, some essential distinctions involved in this concept are explained. Then, in §§ 4–7, some examples of Bocheński's "destruction of superstitions" are presented. It turns out that they are good examples of working against irrational attitudes by means of logical analysis.

Bocheński's fight against superstitions is a part of his contribution to the programme of anti-irrationalism, proposed and realized by the Lvov-Warsaw School (hereafter: LWS), the Polish (or more broadly: the Central-European) branch of analytic philosophy. Bocheński was not a typical member of the LWS as he did not undergo his academic education under the supervision of Kazimierz Twardowski or any of his students. However, one of Bocheński's gymnasium teachers was Zygmunt Zawirski, an early student of Twardowski. Later, in the 1930s, Bocheński intellectually approached Jan Łukasiewicz and his community of Warsaw logicians. In 1936, together with Jan Salamucha and Jan Drewnowski, and with Łukasiewicz's support, they created the so-called Cracow Circle, a group concentrated on reforming Catholic theology by means of formal logic – according to the "syllogism" that since logic is a tool of philosophy, and philosophy, as medieval people used to say, is the *ancilla* (handmaid) of theology, then logic should be a tool of theology.[4] This rather informal and unfortunately short-lived group (it became one more victim of the Second World War) was sometimes considered a branch of the LWS. Even though the contact between Bocheński and the "full" members of the LWS was not institutionalized, Bocheński was deeply touched by the programme and spirit of the School. This is clear not only in his *Formale Logik*, which carries out Łukasiewicz's programme of research in logic in its methodology and history, but even more in Bocheński's general anti-irrationalistic attitude, his drive for clarity of speech and strictness of argumentation, and his emphasis on independent thought. These ideals were implanted in all members of the LWS and form a kind of hallmark of the School.

[4] Ibid., p. 85.

2. What Is Analysis in General and Philosophical Analysis in Particular?

Sto zabobonów. Krótki filozoficzny słownik zabobonów [One Hundred Superstitions: A Brief Philosophical Dictionary of Superstitions] was written by Bocheński with great "moralizing" passion.[5] Bocheński not only analyzes superstitions but also condemns them, and sometimes even makes fun of them. This "moralizing" layer of his dictionary does not interest me here. Therefore, it is worth recalling a few details about the "purely" analytical method that was applied by Bocheński in other works. It should be emphasized that the question of the methods applied in philosophy was very important to him.[6]

To analyze a certain object is to distinguish, with cognitive aims, its components, or properties. We analyze when we seek to answer questions such as "What is it like?," "What are its components?," or "How does it work?" Analysis is a procedure applied in many disciplines, ranging from formal ones (see, for instance, mathematical analysis), through the natural sciences (see, for instance, chemical analysis), to the humanities (see, for instance, literary analysis or musical analysis). One may analyze various kinds of objects, including real, that is, spatio-temporal, objects (for instance, a sample of blood, certain chemical compounds), as well as unreal objects (for instance, mathematical functions, musical compositions taken as composers' ideas).

The choice of analytic methods, namely the procedures that are applied in order to analyze something, is determined by many factors, including the type of object analyzed and the instruments in use. We have direct access to real objects, but we have only indirect access to unreal objects. In a given real thing, for instance, in a given clock, we may manually distinguish its parts, but the components of some non-real objects, for instance the plot of a novel, we may distinguish only mentally. In some kinds of analysis, one needs to use real instruments (for instance, a hammer, a screwdriver, measuring instruments). However, more or less sophisticated conceptual instruments are needed in all kinds of analysis since every analysis is preceded by some theoretical background. Let

[5] J.M. Bocheński, *Sto zabobonów. Krótki filozoficzny słownik zabobonów* [One Hundred Superstitions: A Brief Philosophical Dictionary of Superstitions], Warszawa 1988.

[6] A sketch of Bocheński's views in the domain of the methodology of philosophy may be found in A. Brożek et al., *Anti-Irrationalism: Philosophical Methods in the Lvov-Warsaw School*, forthcoming.

us add that the kinds of elements distinguished in the process of analysis are to a certain degree determined by this theoretical background. A given conceptual scheme indicates the intentional limits of the analysis. For instance, in a given sample imported from Mars, a chemist expects to find elements known from the periodic table. In a piece of music, we usually focus on elements such as tones, motifs, phrases, etc., the kinds of which are established within music theory. It happens, however, that the analyzed objects surprise us and force us to change our initial analytic hypotheses. The sample from Mars may theoretically contain unexpected, unknown elements, and musical compositions may extend the existing analytic schemes.

Philosophers also apply analysis, and not only in the tradition called "analytic philosophy." What is the object of philosophical analysis? Bocheński would probably say that philosophers analyze, first of all, concepts and their systems (conceptual schemes) as well as statements and their systems (theories). A philosopher's aim may be to indicate the components of one concept, to distinguish between two or more concepts or to construct an entire scheme of concepts related to one another. A philosopher may focus on one statement, or sequences of statements (which, for instance, express an argument or reasoning), as well as theories as wholes.

There are various conceptions of what concepts and statements are, how to cognize them, and what kinds of parts or properties could be distinguished in them. Some philosophers are convinced that it is possible to access concepts or statements, or even their "essences," directly. Bocheński's opinion, typical of analytic philosophers, was that the only way to gain access to concepts and statements is via language. Languages are their only intersubjectively accessible carriers. Simplifying, and very roughly speaking, one may say that concepts are (or may be represented as) the meanings of words, and statements are (or may be represented as) the meanings of sentences. Thus, words and sentences (belonging to natural, scientific, or philosophical languages) are the "empirical basis" of philosophical analysis and, at the same time, they serve as a tool which enables us to present the results of analysis. If the results of analysis of concepts and statements are to be expressed, they have to be expressed in a language.

It is obvious that languages, first of all natural languages and the informal languages of the sciences (that is, languages of various disciplines that are not formalized), provide a broad and rich corpus of empirical data, which is, at the

same time, very difficult to process. It is almost never the case that one word is assigned to one concept and one sentence to one statement. In "living" languages, one regularly encounters vague, ambiguous expressions or evolving, fluctuating senses. When one applies analysis to such empirical data, one has to distinguish between and clarify the meanings of words, as well as pick apart and disambiguate the meanings of sentences. Such "corrections" are necessary if we want philosophical language to fulfil the elementary postulates of precision. Thus, due to these imperfections of concepts and statements, philosophical analysis is often combined with corrections or even with the construction of new concepts or conceptual schemes, new statements or new theories. The need for these (re)constructive elements is natural but is not always accepted by practitioners of analysis. In particular, there are currents in analytic philosophy which recommended only the use of natural languages in philosophy, with all of their imperfections. For Bocheński, as for all members of the LWS, philosophical language has to be as precise as possible – so, we cannot operate only in the area of natural languages.

Now, the question arises as to what tools may be used by a philosopher to make these conceptual distinctions or clarify the meanings of sentences. Bocheński belongs to the philosophical tradition in which one looks for the theoretical background of analysis in logic, broadly understood as encompassing theories of formal systems, logical semiotics, and methodology.

Mathematical logic gave philosophical investigations a serious impulse to develop. The most sophisticated way of using formal tools in philosophy consists in the axiomatization of some philosophical theories (or their parts). This procedure equips these theories with a mature form in which axioms and primitive terms are listed, all secondary terms are defined, and all theses are inferred from axioms. Some concepts can be defined, and, finally, inferential steps may be justified in a way that does not raise any doubts. Formal languages may also be used to indicate the logical forms of sentences and make clear the logical relations between sentences, thus ensuring sound reasoning and argumentation.

However, axiomatization may be employed only at the final step of the development of a given theory (including philosophical theories). Before this tool is used, much pre-formal and pre-axiomatic work is required. Here, the elements of logical semiotics, such as the theory of the functions of names, the analysis of semiotic defects, and various kinds of sentence paraphrasing, may be applied. In

the end, the results of analysis are given in definitions of terms that are carriers of concepts and logical paraphrases of sentences that are carriers of statements.

All procedures that make use of broadly understood logical matters are called "logical analysis."[7]

3. What Are Superstitions?

In the introduction to *One Hundred Superstitions*, Bocheński admits to having deliberately used the term "zabobon" instead of the much less pejorative Polish word "przesąd." In English, there are also two terms of similar meaning: "prejudice" and "superstitions," but the emotional elements in the meanings of these terms are perhaps not as strong as in their Polish quasi-equivalents. In this text, as an English equivalent of "zabobon," I choose the English "superstition"; however, let us keep in mind that Bocheński emphasized his contemptuous attitude towards superstitions even by his choice of terminology.[8]

In fact, in *One Hundred Superstitions*, the author assumes, to some extent, the role of moralizer. Some distinctions are introduced vaguely, and some referenced standpoints are "sharpened" in order to show more easily their superstitious character. The more mainstream character of the book, which was written for a larger audience, justifies this style to some degree. Still, the book may serve as a source of examples of applying simple logical tools in philosophical analysis.

Bocheński introduces the following definition:

[7] About Bocheński's conception of analysis, see also M. Lechniak, *J.M. Bocheński's Method of Philosophical Analysis and Contemporary Applied Ontology*, "Studies in East European Thought" 2013, Vol. 65, No. 1–2, pp. 17–26. The methodological peculiarities of the LWS have been recently characterized by M. Będkowski et al., *Analysis – Paraphrase – Axiomatisation: Philosophical Methods in the Lvov-Warsaw School*, in: *Formal and Informal Methods in Philosophy*, eds. M. Będkowski et al., Leiden 2020, pp. 56–74.

[8] He commented on this as follows: "Someone would say that by using this vituperative term ["zabobon"] I offend the venerable principles of polite comradeship. For, in the world of philosophers it is customary to deal elegantly with even the worst idiocy. When one wiseacre states that there is no world, or that it exists only in his head; when the second wiseacre proves that I cannot be sure that I am sitting right now, and when the third tells us that we have no consciousness or feelings – it is said to be a 'view,' 'opinion,' 'philosophical theory' and it is taught venerably to students" – J.M. Bocheński, *Sto zabobonów*, op. cit., p. 8.

(3) I define "superstition" as follows: a belief which is (1) obviously and to a high degree false but still (2) considered surely true.[9]

In the first step, I propose modifying two elements of this formula. Firstly, I am convinced that "statement" is a better *genus* for the definition of "superstition" than "belief."[10] Secondly, I would exchange "obviously and to a high degree false" with simply "obviously false" as I do not know what "levels of truthfulness" could be (and unfortunately I cannot ask Bocheński what he had in mind when using this expression). That is why the point of departure should be the formula:

(4) A superstition is a statement which is obviously false and still accepted as true.

Formula (4) requires further and deeper comments. Here, I will limit myself to three such comments.

Firstly, it is assumed here that one should distinguish the truthfulness of statement *S* from accepting *S* as true (or, in other words, being convinced that *S*). Take the statement expressed in the sentence "Bocheński was a Pole" as an example. Let us agree that this statement is true even if there is someone who does not accept this statement or even rejects it. Let us now consider a set of someone's convictions, and let this be the set of Bocheński's convictions in 1920, assuming that these convictions may be represented as sentences. Well, it is easy to guess that in the set of sentences accepted by Bocheński in 1920 there were some true

[9] Ibid., p. 7.

[10] Dictionaries often provide definitions of "superstition" in which, like in Bocheński's book, the *genus proximum* is the term "belief" (cf., e.g., *The New Penguin English Dictionary*, ed. R. Allen, London 2001). However, the phrase "a belief in superstition(s)" or "to believe in superstition(s)" is in common use (cf., e.g., *The Encyclopedia of Superstitions*, ed. R. Webster, Woodbury, MN 2008, p. x). Now, if we substitute the term "superstition" with the *definiendum* of the definition of "superstition" as a belief in these phrases, we get: "superstition = a belief in a certain belief." Taking "statement" as the *genus* solves this problem. It is significant that, for example, in *Mały słownik języka polskiego* [Little Dictionary of the Polish Language], ed. E. Sobol, Warszawa 2020, "[człowiek] zabobonny," that is, "superstitious [person]," is defined as "[człowiek] wierzący w zabobony," that is, "[person] believing in superstitions," and "superstition" is defined as "a [certain] belief" (just like in the English dictionary cited above). Unfortunately, it often happens that dictionaries lack logical culture. Let me also note on the margin that introducing the distinction between the concept of "superstition" and the concept of "belief in superstition" makes it easier to separate the logical issues (of superstitions as statements) that interest me – from the psychological ones (of belief in superstition), which was mentioned by one of the reviewers of this paper, and which certainly belongs to an interesting but different research field.

as well as some false sentences. However, only some of these false sentences are superstitions. Being false and being accepted are not sufficient conditions of being a superstition.

Definition (4) assumes, secondly, a distinction between falsity *simpliciter* and obvious falsity. Under the entry for "superstition," Bocheński explains what "obviously false" means by listing the following sources of obvious falsity:

> (5) An obviously false statement is either meaningless or blatantly inconsistent with facts, or inconsistent with the laws of logic.[11]

Again, I cannot restrain myself from a little intervention in that list. In my opinion, if a statement is meaningless, it is not a statement *sensu stricto*, so it is neither true nor false. That is why I would limit "obvious falsehood" to Bocheński's other two examples: contradiction (inconsistency with the laws of logic) and blatant inconsistency with facts.

Thirdly, let it be noted that an obviously false statement is a superstition only as long as it is accepted by someone. Let us agree that in order to accept a statement (or reject it) in the proper sense, one has to understand it. So, the problems of superstitions do not concern mad or unintelligent people who are unable to understand language expressions. Bocheński emphasizes that some obviously false statements happen to be accepted by people who are brilliant… Sometimes even millions of such people can accept a certain obviously false statement. How can this be? Nobody would like to accept an obviously false statement. People seem not to want to accept a sentence they know to be obviously false. So, the problem is that these people, despite their brilliance and intelligence, do not see the obvious falsity of some of their convictions and are not aware of it.

Bocheński admitted that in his adolescence he himself had been a victim of many superstitions, which, however, he later condemned – he freed himself from them. This is what a rational person does: refutes false beliefs upon realizing their falsity. It is not always easy to part with a conviction which is "deeply" accepted or to which we are emotionally attached. Sometimes we need somebody to show us the obvious falsity of a statement we accept.

After these explanations, it is easy to see in what sense logical analysis serves to destroy superstitions. The analysis of a given statement or a given concept involved in a statement makes it easier to reveal that this statement is obviously

[11] J.M. Bocheński, *Sto zabobonów*, op. cit., p. 115.

false. And, for a rational person, recognizing a certain sentence as obviously false is a sufficient reason to refute that sentence, to get rid of a superstition. The social role of analytic philosophers becomes clear: they make people realize that some statements accepted by them are false and should be rejected. Bocheński emphatically calls this process of freeing oneself from superstitions "spiritual resurrection."

4. Superstitions about Gibberish

From the fact that superstitions are (meaningful) statements, it does not follow, of course, that "superstitious" statements cannot concern gibberish. It's no wonder that Bocheński gives examples of just such superstitious gibberish.

A relatively harmless kind of gibberish is uttering words which are completely devoid of sense (like "hocus-pocus"). According to Bocheński, what is more dangerous is the kind of gibberish that consists in uttering words which have meaning but which are connected in a way that does not form a meaningful sequence of words. This form of gibberish Bocheński calls "abuse of the meaning of words." He accuses some theologians and philosophers of applying it and presenting their "erudite gibberish" as "deep truths."

Whoever takes such "erudite gibberish" for a meaningful expression, is a victim of a superstition.

> Whoever considers gibberish to be a means of communicating objective information is a victim of superstition. […] Believing that it may be useful for anything in this field is a gross superstition. Therefore, also the belief that a philosopher can or even should use gibberish, is a superstition. […] The use of words that have subjective meaning is a specific characteristic of humans (and probably, at least partly, of higher animals). Those who would like to replace objective, comprehensible, human speech with gibberish bring humans down to a level lower than that of monkeys and rhinoceroses, because even those beasts use more than gibberish.[12]

[12] Ibid., pp. 24–25.

5. Superstitions Concerning Authority

Let us now concentrate on some of Bocheński's analyses of superstitions included in the aforementioned book.

Bocheński's main work on authority is his monograph *Was ist Autorität?*, published in German in 1974 and later translated into French and Polish.[13] Since Bocheński's analysis of superstitions refers to the content of this book, let me recall its main points.[14]

Bocheński's starting point consists in analyzing some expressions in which the term "authority" occurs. He takes some natural-language statements as well as some theoretical approaches. This empirical material appears to be a real tangle of meanings. Even the category of authority is not easy to establish as it happens that authority is considered to be individual (someone is an authority), a property (someone has authority) or a relation (someone is an authority for someone else). This proves that some reordering is needed; the concept of authority requires a reconstruction. Bocheński decides that authority is basically a relation and, moreover, is convinced that it is a ternary relation between a certain object, a certain subject, and a certain domain. Objects and subjects of authority are conscious human beings, while the domain of authority may be represented as a set of sentences.

Bocheński notes that when one calls someone an authority (without any additions) or even when one says that person *A* is an authority over person *B* (without any additions), there is something missing in such a statement: it is elliptical. The concept of authority is entangled in a ternary relation and when speaking about authority, one has to recognize who the authority is, for whom, and in what domain. For instance, chemistry teachers are authorities for their students in the domain of chemistry (but not, for example, in the domain of the history of literature).

This simple observation helps us get rid of many superstitions. Bocheński writes:

[13] In English, Bocheński published a shortened paper (J.M. Bocheński, *An Analysis of Authority*, in: *Authority*, ed. F.J. Adelmann, The Hague 1974, pp. 56–85).

[14] Detailed reconstructions of the concepts of deontic authority and epistemic authority may be found in A. Brożek, *Bocheński on Authority*, "Studies in East European Thought" 2013, Vol. 65, No. 1–2, pp. 115–133.

> [One of the superstitions] connected with authority is the conviction that there are, so to speak, universal authorities, that is, people who are authorities in all domains. This is obviously not the case; each person is only an authority in a definite discipline. […] Unfortunately, accepting such universal authorities is a very widespread superstition. When, for instance, a group of university professors signs a political manifesto it is assumed that the readers of such a manifesto would consider them authorities in the domain of politics, which they obviously are not […]. For these professors are surely authorities in the domains of [the history of] the French revolution, Chinese ceramics, or probability calculus but not in the domain of politics, and by signing such a manifesto they abuse their authority.[15]

Now, even more than in Bocheński's times, we are often induced to believe that famous scholars or actors are authorities in the domain of morality or politics. A famous scholar or a famous actor, he argues, is certainly an authority in the domain of physics or acting, respectively, but they do not automatically become specialists in the current political situation, and they are not necessarily good advisers regarding our ethical choices. Let us repeat: there are no universal authorities and those who believe in them are just allowing themselves to be manipulated by false authorities.

Bocheński's most famous contribution to the theory of authority is his clear distinction between epistemic and deontic authority. In the case of epistemic authority, that is, the authority of an expert, the domain of authority is a set of "theoretical" sentences. Assume that *A* and *B* are humans and *D* is a set of sentences. Now, we may define "epistemic authority" as follows:

(6) *A* is an epistemic authority for *B* in domain *D*, iff *B* accepts, in principle, as true every sentence which was communicated to *B* in the affirmative by *A* and which belongs to domain *D*.

Deontic authority is the authority of a boss. The domain of this kind of authority is a set of orders. In particular:

(7) *A* is a deontic authority for *B* in domain *D*, iff there is an event *E* such that (1) *B* desires *E* to be realized and (2) *B* is convinced that *B*'s

[15] J.M. Bocheński, *Sto zabobonów*, op. cit., p. 21.

> execution of all orders given by *A* with emphasis, and belonging to domain *D*, is a necessary condition for the realization of *E*.

Bocheński emphasizes that these two kinds of authority should not be confused and whoever takes a deontic authority for an epistemic one or vice versa is engaged with a superstition. It may happen that our bosses are also experts but it is not always the case. Bocheński comments that:

> Lots of people believe that the one who has power, and thus is a deontic authority, is at the same time an epistemic authority, who can instruct his subjects, for instance in astronomy. […] Outstanding people are also victims of this superstition, for instance Ignatius of Loyola, the founder of the Jesuit Order, in a famous letter to the Portuguese Fathers demanded that they "submit their reason to their superior," that is, to a purely deontic authority.[16]

These two superstitions have been described in order to protect us from the overuse of authority: we should be careful in accepting someone as an authority, we should not follow "false authorities," and we should clearly distinguish experts from bosses. Of course, reliance on authorities should be accompanied by cognitive caution. However, the next warning regarding superstition prevents us from refuting all authorities. Bocheński observes that we live in an era of a fast development of science and specialization. No one is able to grasp all domains of all disciplines. This is, as Bocheński would say, simply a fact. This is why we are doomed to depend on epistemic authority. The same concerns deontic authorities. There are situations in which it is reasonable to follow someone else's instructions. Consider the radical example of a sinking ship on which it is reasonable to follow the orders of the captain. Thus, Bocheński argues, refuting all authorities is simply contradictory to facts, and thus it is a superstition that it could be done.

> To follow an authority is often a very rational attitude, in accordance with reason. When, for instance, a mother tells her child that there is a great city called "Warsaw," then the child is quite rational in accepting this as truth. Similarly, a pilot acts reasonably when he follows the instructions of a meteorologist who states that there is high air pressure in Warsaw and wind from the west […] since there is information in both cases beyond the knowledge of the child or the pilot, respectively. We also use authority in science. In order to believe

[16] Ibid., pp. 21–22.

> this, it is enough to look at the huge libraries of any scientific institute. Books in these libraries contain the reports of the results of other sciences, so they are statements of epistemic authority.[17]

Thus, the statement that there is always and everywhere a contradiction between authority and reason is, so to speak, a meta-superstition.

6. Superstitions Connected with Patriotism

The next example of superstitions relates to the concept of patriotism. The term "patriotism" in ordinary language causes similar problems to the term "authority." It is used with different, fluctuating meanings. Moreover, it is often the case that the participants of politics or social discussions manipulate these meanings. The only solution is, again, to introduce some conceptual ordering.

Bocheński's terminological adjustment may be expressed in the following definition:

(8) *A* is a patriot, iff *A* loves *A*'s country and *A*'s countrymen.[18]

Thus understood, patriotism is, as Bocheński emphasizes, "not a superstition but a virtue."

The question arises of how to operationalize the love of one's country and countrymen. Some of Bocheński's comments suggest that to love your country and your countrymen means at least to want what is good for them.

Now, the first superstition connected with patriotism is to confuse it with nationalism. In order to see the source of this confusion, we need a definition of "nationalism." The following formula reflects Bocheński's intentions concerning this term:

(9) *A* is a nationalist, iff *A* adores/idolizes *A*'s nation and hates other nations.

Once the definitions are established, the source of superstition becomes clear. Based on these assumptions, it is obvious that from the sentence in the form

[17] See ibid., p. 21. In reconstructions of Bocheński's definitions or quasi-definitions I omit quantifiers.

[18] Ibid., p. 81.

"*A* is a patriot" the sentence "*A* is a nationalist" does not follow. Firstly, "idolizing" is something more than loving (let us set aside the question of how to operationalize these predicates). Secondly, from the fact that *A* loves object *X*, it does not follow that *A* does not love object *Y*. Thus, one who conflates patriotism and nationalism is guilty of a superstition.

The confusion of patriotism and nationalism has two sides, that is, it leads to two kinds of irrational attitudes. On the one hand, those patriots who become nationalists fall victim to a superstition. On the other hand, those who condemn patriotism because they unconsciously transform it into nationalism are also victims of superstition. According to Bocheński, the second superstition is more widespread and more dangerous.

> [This means] that whoever loves their country more than, for instance, Ecuador or Vietnam is accused of racism. Moreover, if someone faced with a choice gives priority to their own countrymen over foreigners, that person is considered a racist criminal, like a Hitlerite.[19]

According to Bocheński:

> Every human being has the right to take care of, first of all, people that are related to them, and without any thought about the superiority of this or that race.[20]

7. Superstitions Related to the Concept of Tolerance

Bocheński characterizes tolerance as "[the act of] enduring" ["znoszenie"]. This requires some clarification. Let us take the following formula as the point of departure:

(10) *A* tolerates action *X* iff *A* endures action *X*.

I guess that Bocheński would agree that this formula may serve as the most basic explication of what tolerance is. He wrote that "we call a person 'tolerant' if they endure others, their views, their style of life."[21]

[19] Ibid.
[20] Ibid.
[21] Ibid., p. 109.

The concept of enduring requires further analysis. Let me only state shortly that it seems that the meaning of "enduring" may have two sides: behavioural and emotional. To endure something means not to act against it or to have at least a neutral emotional attitude towards it (not to condemn it).

(11) *A* tolerates action *X* iff *A* does not act against action *X*.

(12) *A* tolerates action *X* iff *A* does not condemn action *X*.

Let us emphasize this firmly: to endure a certain action does not mean to support this action or to affirm this action. Today, perhaps more than in Bocheński's times, we are asked to support or affirm some behaviours in the name of tolerance. Such requirements surely go beyond simple enduring.

Now, a tolerant person is a person who tolerates the actions of other people. A question arises: which actions? All of them or only the majority? It seems that tolerance is a gradable property. In such a situation, we may only define maximal or relative tolerance. These concepts, in their simplest versions, may be defined as follows:

(13) *A* is maximally tolerant iff *A* tolerates all actions.

(14) *A* is more tolerant than *B* iff *A* tolerates more actions than *B*.

The principle of tolerance is connected with the concepts of tolerance and tolerant people. Superstitions arise precisely when it comes to this principle. What should it state? Bocheński emphasizes that generally tolerance is "a good way for various groups within one society, which differ with respect to worldviews or basic political theses, to coexist."[22] However, accepting the postulate of tolerance without any limitations is also a superstition. The postulate:

(15) Everyone should be maximally tolerant. / Every action should be tolerated.

should not be accepted by a rational person.

Suppose that action X_1 consists in offending us or our relatives. The postulate of maximal tolerance would require us not to act against such an offence (let as leave aside the emotional interpretation of tolerance as less important here).

[22] Ibid.

According to Bocheński, such actions, actions of violence, should obviously be excluded from the domain of the principle of tolerance.

The second example concerns political life. Suppose that action X_2 consists in the removal of the principle of tolerance from the regulations of a given country. In the name of the principle of tolerance, should we tolerate such attempts? According to Bocheński, obviously not. He states:

> Then there are two superstitions: no tolerance entitles anyone to offend others, and a tolerance that tolerates its own enemies cannot stand.[23]

The statement that the principle of tolerance should not have any limitations is a superstition.

Bocheński drew attention to one more superstition connected with tolerance, the consequence of which is the destruction of the world of scholarly activities. According to the principle of tolerance (and, let us add, the principle of freedom of thought), no beliefs are prohibited. However, it is not the case that in the name of tolerance every kind of research has an equal right to be institutionally or financially supported. Bocheński's example is the following:

> If there were someone defending the system of Ptolemy, then in tolerant countries this would not be prohibited. However, most likely such a person could not find an institute of astronomy that would finance such "research."[24]

Bocheński is aware that revolutions in science take place when some essentially new theories are discovered. However, generally speaking, tolerance in science should be limited in cases where the standards of scientific methods are abused.

8. Closing Remarks

Let us recapitulate the main points of the paper.

Bocheński believed that destroying superstitions, that is, obviously false but accepted statements, is one of the practical tasks of philosophers. From the methodological point of view, he was an analytic philosopher; therefore, he made use of the logical tools of formal logic, logical semiotics, and methodology in

[23] Ibid.
[24] Ibid.

order to analyze concepts and their schemes, statements, or whole theories. By distinguishing between and clarifying the senses of terms and providing logical paraphrases of sentences, one may demonstrate the obvious falsity of a given statement. For a rational human being, to see the obvious falsity of a statement is a sufficient condition for rejecting this statement, for parting with a superstition.

Bocheński's *One Hundred Superstitions* contains simplified analyses of some superstitions encountered in contemporary societies. As one may see based on the reconstructed examples (authority, patriotism, tolerance), the logical tools used by Bocheński are quite simple. Usually, he limits himself to definitions that clarify the meanings of words, or he disambiguates sentences by providing a quantifier. Still, these simple tools serve to make the reader realize the "superstitiousness" of some widespread statements.

By means of his programme of freeing people from superstitions, Bocheński joined the more general programme of anti-irrationalism realized in the LWS. Certainly, Bocheński was the kind of person described by Łukasiewicz:

> It is as if the scales fall from the eyes of whoever has educated themselves in the exact thinking of mathematical logic. They see differences where others do not, and they see nonsense where others look for some mysterious depth.[25]

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[25] J. Łukasiewicz, *O znaczeniu i potrzebach logiki matematycznej* (1929) [On the Importance and Needs of Mathematical Logic], in: J. Łukasiewicz, *Logika i metafizyka* [Logic and Metaphysics], ed. J.J. Jadacki, Warszawa 1998, p. 431.

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Summary

According to Józef M. Bocheński, philosophers should analyze and not moralize. Bocheński also wrote that the philosophers' aim is to battle with superstitions. The present paper concerns the question of how philosophical analysis may serve this aim. In the first part of the paper, the kinds and tools of philosophical analysis are presented. In particular, the objects of philosophical analysis (concepts and statements) as well as logical "instruments" of analysis are discussed. In the second part, the concept of superstition (or prejudice) is analyzed. Taking Bocheński's definition as the point of departure, some essential distinctions in-

volved in this concept are explained, for instance the distinction between truthfulness of a statement and somebody's acceptance of a statement, and the distinction between a false statement and an obviously false statement. Next, the mechanism of fighting against superstitions by means of analysis is shown using examples taken from Bocheński's book *Sto zabobonów* [One Hundred Superstitions] (1987). Superstitions connected with authority, patriotism, and tolerance are analyzed.

Key words: authority, logical analysis, patriotism, superstitions, tolerance

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J.M. Bocheński's Logical Analysis of Kant's Criticism of the Cosmological Argument

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Introduction

Since its publication, Immanuel Kant's *Critique of Pure Reason* has continuously been subject to numerous analyses, conducted both by those who admire the genius of the philosopher from Königsberg and by those who disagree with his views. The methods used in these studies vary considerably and encompass the tools of logic. This type of analysis was undertaken by Father Józef Maria Bocheński (1902–1995), who called himself "a converted Kantist," but who was also an expert on logic.[1]

The beginning of the 20th century was a time of a dynamic development of mathematics and logic. The new approach to logic was named logistics in order to distinguish it from the traditional approach. It was applied in the analysis of philosophical views. Such was the goal of the creators of the Lvov-Warsaw School. Those ideas were also employed in the field of Christian philosophy by the members of the so-called Cracow Circle.[2] Its aim was to apply the tools of contemporary logic in the fields of Christian philosophy and theology. The Circle

[1] J.M. Bocheński, *Między logiką a wiarą. Z Józefem M. Bocheńskim rozmawia Jan Parys*, Warszawa 1998, p. 27.

[2] The term "Cracow Circle" was introduced into scientific discourse rather late. It was first used by Bocheński only in his article of 1987. Cf. J.M. Bocheński, *Koło Krakowskie*, "Kwartalnik Filozoficzny" 1995, Vol. 23, No. 1, pp. 23–31; J.M. Bocheński, *The Cracow Circle*, in: *The Vienna Circle and the Lvov-Warsaw School*, ed. K. Szaniawski, Dordrecht 1988, pp. 9–18. In all his earlier

formed at the beginning of 1934. It was created by Bocheński together with Father Jan Salamucha (1903–1944), Jan F. Drewnowski (1886–1978), and Bolesław Sobociński (1906–1980). The specific goals of the programme established by the Circle, as stated by Bocheński, can be expressed by means of the following postulates: "(1) to make philosophers and theologians use the appropriate scientific language; (2) to make them use modern formal logic, as well as semiotic and methodological concepts instead of scholastic terminology; (3) to make them use formalism."[3] An important event promoting these ideas was a special session organized during the Third Polish Philosophical Congress in Cracow in 1936.[4] Members of the Circle wanted to introduce the style of philosophizing typical of the Lvov-Warsaw School into Catholic thought. Their works from that period concerned the *ex motu* proofs from Thomas Aquinas's *Summa contra gentiles*, his argument for the immortality of the soul, the scholastic concept of analogy, and the history of medieval logic. It is worth noting that the creation of the Circle was influenced not only by the development of logic and the assimilation of the postulates of the Lvov-Warsaw School, but also by the spirit of that era (optimism and bravery in realizing great projects in various areas of life) and the contemporary situation of theology (the Catholic Church versus modernism).[5] Despite the fact that the Circle's activity ceased with the outbreak of the Second World War, its aims seem to be valid also today.[6]

At the end of his life, Bocheński returned to pursuing those goals and the results of his work were presented in the book *Gottes Dasein und Wesen. Logische*

publications, as well as in the works of the other members of the Circle and of their mentor, Father Professor Konstanty Michalski, this term is never used.

3 J.M. Bocheński, *Wspomnienia*, Kraków 1994, pp. 123–124. Unless stated otherwise, all translations are my own.

4 Presentations and discussions from this session were published in *Myśl katolicka wobec logiki współczesnej*, ed. J. Salamucha, "Studia Gnesnensia" 1937, Vol. 15.

5 Cf. M. Tkaczyk, *Geneza Koła Krakowskiego*, "Studia Philosophiae Christianae" 2019, Vol. 55, No. 2, pp. 9–39.

6 More information concerning the Cracow Circle and its programme can be found in the following publications: J.M. Bocheński, *O metodzie teologii w świetle logiki współczesnej*, "Collectanea Theologica" 1949, Vol. 21, pp. 171–192; Z. Wolak, *Neotomizm a szkoła lwowska-warszawska*, Kraków 1993; Z. Wolak, *Zarys historii Koła Krakowskiego*, in: *Logika i metafilozofia*, ed. Z. Wolak, Kraków 1995, pp. 79–84; J. Woleński, *Ontologia w Kole Krakowskim*, in: *Logika i metafilozofia*, ed. Z. Wolak, Kraków 1995, pp. 85–98; R. Murawski, *Cracow Circle and Its Philosophy of Logic and Mathematics*, "Axiomathes" 2015, Vol. 25, pp. 359–376; M. Tkaczyk, *Cracow Circle: Theology in the Lvov-Warsaw School*, in: *The Significance of the Lvov-Warsaw School in the European Culture*, eds. A. Brożek, F. Stadler, J. Woleński, Wien 2017, pp. 173–188.

Studien zur Summa Theologiae I, qq. 2–11.[7] Unfortunately, the book had not been authorized before it was published, and the corrections added by its editors were not always marked. For that reason, the manuscripts of this study are of great significance when it comes to the analysis of its content. The structure of *Gottes Dasein und Wesen* is based on the analysis of questions 2–11 from Aquinas's *Summa theologiae*. The analysis finishes with a chapter concerning the possibility of creating an axiomatic theory of the Absolute on the basis of the results of the analysis. The research is placed in a broader context of Bocheński's programme of studies on God. In point 5, Bocheński writes that "a critical analysis of Kantian and neopositivist objections to the possibility of knowing God and to the proofs of his existence is an urgent task."[8] In order to pursue this goal, in his *Gottes Dasein und Wesen*, Bocheński undertakes that kind of research. It is included in Chapter 8, labelled as an appendix. His studies focus on the criticism of the cosmological argument for God's existence, put forward by Kant in his *Critique of Pure Reason*.[9] The necessity of conducting this kind of research was one of the subjects of Bocheński's lecture delivered in 1990 in Warsaw, during the ceremony of awarding him the *doctor honoris causa* degree at the Academy of Catholic Theology.[10] The present paper aims to critically present Bocheński's less-known analyses concerning Aquinas's proofs of God's existence against the background of those more popular ones. The source text used by the author of this study is available in two versions: in book form (B) and in its Polish manuscript (M).[11]

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7 J.M. Bocheński, *Gottes Dasein und Wesen. Logische Studien zur Summa Theologiae I, qq. 2–11*, München 2003. The book contains amended analyses which were originally published in the article *Die fünf Wege*, "Freiburger Zeitschrift für Philosophie und Theologiae" 1989, Vol. 36, No. 3, pp. 235–265 (in Polish: *Pięć dróg*, trans. J. Miziński, in: J.M. Bocheński, *Logika i filozofia. Wybór pism*, Warszawa 1993, pp. 471–473). Additionally, there exist two manuscripts of the book: in German and in Polish, from 1989 and 1993, respectively.

8 In the Polish manuscript, Bocheński writes about objections to the proofs of God's existence, while in the German manuscript and his book he writes about doubts concerning knowing God.

9 I. Kant, *Critique of Pure Reason*, trans. and eds. P. Guyer, A.W. Wood, Cambridge 1999, pp. 569–575. Arguments for the existence of God were analyzed by Kant also in his work titled *The Only Possible Argument in Support of a Demonstration of the Existence of God* (1763). In his *Critique of Pure Reason*, we can find references to those earlier investigations from the period, which Kant later called "dogmatic slumber."

10 Cf. J.M. Bocheński, *O współczesnym stanie i zadaniach teologii filozoficznej*, "Studia Philosophiae Christianae" 1991, Vol. 2, pp. 103–107.

11 The table of contents of the available German manuscript refers to a fragment concerning Kant, but, unfortunately, it is not included in the text. A comparison of the previous parts of the Polish and German manuscripts shows that there are no significant differences between them when it

The Stages of Analyzing Kant's Text

The way Bocheński conducted his analysis of Kant's text resembles the way he arranged his logical commentary on the *Summa theologiae*. First, he identifies the essential fragment of the discourse of a given work, divides it into separate sentences, numbers them and then formalizes them. Next, he analyzes the specific arguments in order to check whether there are any fallacies in them. The focus is on the truth of the applied premises (material fallacy) or their acceptability on the grounds of a given philosophical system and the logical consequence of the premises and the conclusion (formal fallacy).

Bocheński focuses only on the criticism of the cosmological argument as he believes that the only valid way among the Thomist ways is the one that concerns the efficient cause, that is, the second way. It represents the type of argument referred to as cosmological. Bocheński chooses not to discuss Kant's criticism of the ontological argument because Aquinas rejects that kind of argument too.

The fragments of interest are divided by Bocheński into shorter sections and then analyzed. These include:

- the report on the cosmological argument;
- the reduction of the cosmological argument to the ontological argument;
- the four "simplifications" included in the cosmological argument.

The Applied Abbreviations and Schemas of Reasoning

The formalization is conducted with the help of the following abbreviations:

BS(x, y) =: *x ist durch den Begriff von y bestimmt (x is described by concept y)*,[12]
CA(x, y) =: *y ist die Ursache von x (y is the cause of x)*,
E!(x) =: *x existiert (x exists)*, [in (M) we have: *E(x)*]
En!(x) =: *x existiert notwendigerweise (x exists out of necessity)*,
[in (M) we have: *En(x)*]

comes to the content of the analysis and the formalization of the studied text. Thus, it can be assumed that in this situation there will be no differences either.

[12] In brackets I provide my translations.

Eb!(x) =: *kann nur auf eine einzige Art bestimmt werden (x can be described only in one way)*, [in (M) we have: *Eb(x)*]
Ber(x) =: *x ist der Begriff des entis realissimi (x is an entis realissimi concept)*, [in (M) we have: *Er(x)*]
H(x) =: *x ist das höchste Wesen (x is the highest being)*, [in (M) we have: *D(x)*]
i =: *ich (I)*,
N(x) =: *x ist ein notwendiges Wesen (x is a necessary being)*,
P(x) =: *x ist kontingent (x is contingent)*, [in (M) we have: *x ist zufällig (x is contingent)*]
Rm(x) =: *x ist ein allerrealstes Wesen (ens realissimum) (x is the most real being (ens realissimum))*,[13]
G(x) =: *x ist gültig (x is valid)*,[14]
W(x) =: *x ist wahr (x is true)*,
k =: *kosmologischen Gottesbeweis (the cosmological argument)*,
o =: *ontologischen Gottesbeweis (the ontological argument).*

Apart from the rule of substitution,[15] Bocheński applies the following rules of reasoning:[16]

g $\dfrac{p \to q \quad p}{q}$ j $\dfrac{\underset{x}{\forall}[\Phi(x) \to \underset{y}{\exists}\Psi(x, y)] \quad \underset{x}{\exists}\Phi(x)}{\underset{x}{\exists}\underset{y}{\exists}\Psi(x, y)}$ m $\dfrac{\underset{x}{\forall}[\Phi(x) \to \Psi(x)] \quad \underset{x}{\exists}\Phi(x)}{\underset{x}{\exists}\Psi(x)}$

[13] In the table of abbreviations on p. 167, which, apart from this exception, is identical with what is included at the beginning of the analysis, we read *Rm(x)* =: *x ist ein ens realissimum*.

[14] The last four abbreviations are not given by Bocheński in the list of abbreviations, but provided later, in his formalization of Kant's text.

[15] Bocheński uses the rule of substitution not only for individual variables, but also within predicates, which shows that he uses second-order logic.

[16] Rules α and β were provided by Bocheński only in the chapter concerning Kant's text, and not with the other rules in Chapter 2 of his book, which constitutes an introduction to the analyses presented in the whole work. That list lacks any references to the discussed fragment concerning Kant, which may mean that the text was written later than the one concerning Aquinas's *Summa theologiae*.

$$\text{n}\quad \begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array} \qquad \text{o}\quad \begin{array}{l} p \to q \\ \sim q \\ \hline \sim p \end{array} \qquad \alpha\quad \begin{array}{l} \Phi(a) \\ \hline \exists_x \Phi(x) \end{array} \qquad \beta\quad \begin{array}{l} \exists_x [\Phi(x) \wedge \Psi(x)] \\ \hline \exists_x \Psi(x) \end{array}$$

The procedure of identification of the applied rules of reasoning presented above is used by Bocheński to reconstruct the argument structure of the analyzed text. Obviously, when we are familiar with those rules, we can identify the type of logical calculus (or its fragment) used. However, Bocheński does not elaborate on this issue here. We can find remarks of that type, concerning argumentation, in his analyses of the *Summa theologiae*.

The Report on the Cosmological Argument

The text we are interested in is divided by Bocheński into separate sentences and numbered as shown below:[17]

No.	German version	English translation
1.1	Wenn etwas existiert, so muß auch ein schlechterdings nothwendiges Wesen existieren.	If something exists, then an absolutely necessary being also has to exist.
1.2	Nun existiere zum mindesten ich selbst:	Now I myself, at least, exist;
1.3	also existiert ein absolut nothwendiges Wesen.	therefore, an absolutely necessary being exists.
1.4	Diese Schlußfolgerung beruht auf dem vermeintlich transcendentalen Natur gesetz der Causalität: daß alles Zufällige seine Ursache habe	It rests on the allegedly transcendental natural law of causality that everything contingent must have a cause,
1.5	die, wenn sie Wiederum zufällig ist, eben sowohl eine Ursache haben muß,	which, if it in turn is contingent, must likewise have its cause,
1.6	bis die Reihe der einander untergeordneten Ursachen sich bei einer schlechthin nothwendigen Ursache endigen muß,	until the series of causes subordinated one to another has to end with an absolutely necessary cause,
1.7	ohne welche sie keine Vollständigkeit haben würde.	without which it would have no completeness.

[17] The German text is quoted from Bocheński's book (the version from the manuscript is slightly different). The English text is quoted from: I. Kant, *Critique of Pure Reason*, op. cit., p. 570.

In order to improve the disquisition presented above, Bocheński formulates the following premise, which is accepted by Kant tacitly:

No.	German version	English translation
1.21	Wenn ich existiere, dann existiert etwas.	I exist, then something exists.

Reconstruction:

Premises:

1. (M) $\underset{x}{\forall}\{\underset{y}{\exists} CA(x, y) \rightarrow \underset{z}{\exists}\underset{t}{\forall}[CA(z, t) \wedge N(t)]\}$

   For each x, if for a certain y, y is the cause of x, then for a certain z, for every t, t is the cause of z, and t is a necessary being.

   (B) $\underset{x}{\forall}[\underset{y}{\exists} CA(x, y) \rightarrow \underset{z}{\exists} N(z)]$

   For every x, if for a certain y, y is the cause of x, then for a certain z, z is a necessary being.

2. $\underset{x}{\forall}[P(x) \rightarrow \underset{y}{\exists} CA(x, y)]$

   For every x, if x is contingent, then for a certain y, y is the cause of x.

3. (M) $E(i) \wedge P(i)$

   (B) $E!(i) \wedge P(i)$

   I exist and I am contingent.

Proof:

4. (M)	$\underset{x}{\forall}\{P(x) \rightarrow \underset{z}{\exists}\underset{t}{\forall}[CA(z, t) \wedge N(t)]\}$	1, 2, Barbara
(B)	$\underset{x}{\forall}[P(x) \rightarrow \underset{z}{\exists} N(z)]$	1, 2, Barbara
5. (M)	$\underset{x}{\exists}[E(x) \wedge P(x)]$	3, α
(B)	$\underset{x}{\exists}[E!(x) \wedge P(x)]$	3, α
6.	$\underset{x}{\exists} P(x)$	5, β

7. (M) $\exists_z \forall_t [CA(z, t) \wedge N(t)]$ 4, 6, j

(B) $\exists_z N(z)$ 4, 6, m

The formalism presented above is considered correct by Bocheński. Indeed, the rules of reasoning are used here correctly. In line 7 from version (B) a different rule is used from the one provided in the formalization. However, the problem lies in the formulation of premise 1. That problem is discussed by the editors of (B) in footnote 51. They modify that premise since they believe that it was distorted during the editorial procedure, and that probably its original form was the following: $\forall_x \{P(x) \rightarrow \exists_z \forall_t [\sim CA(z, t) \wedge N(z)]\}$ or $\forall_x \{P(x) \rightarrow \exists_z \{N(z) \wedge \forall_t [P(t) \rightarrow CA(t, z)]\}$. The premise taken from (B) results both from the first and the second supposed form on the grounds of classical logic. Thus, this fragment of Bocheński's formalization requires corrections and improvements.

Reducing the Cosmological Argument to the Ontological Argument

Bocheński formalizes the following text:[18]

No.	German version	English translation
2.1	Das nothwendige Wesen kann nur auf eine einzige Art, d.i. in Ansehung aller möglichen entgegengesetzten Prädicate nur durch eines derselben, bestimmt werden,	The necessary being can be determined only in one single way, i.e., in regard to all possible predicates, it can be determined by only one of them,
2.2	folglich muß es durch seinen Begriff durchgängig bestimmt werden.	so consequently it must be thoroughly determined through its concept.
2.3	Nun ist nur ein einziger Begriff von einem Dinge möglich, der dasselbe *a priori* durchgängig bestimmt, nämlich der des *entis realissimi.*	Now only one single concept of a thing is possible that thoroughly determines the thing *a priori*, namely that of an *ens realissimum*.
2.4	Also ist der Begriff des allerrealsten Wesens der einzige, dadurch ein nothwendiges Wesen gedacht werden kann,	Thus the concept of the most real being is the only single one through which a necessary being can be thought,
2.5	d.h. es existiert ein höchstes Wesen nothwendiger Weise.	i.e., there necessarily exists a highest being.

[18] The English text is quoted from: I. Kant, *Critique of Pure Reason*, op. cit., p. 570.

In Bocheński's views, here we are dealing with a particularly complicated and flawed text. To him, there are too many premises accepted tacitly, which clashes with the greatness and rank of such a philosopher as Kant. The premises are as follows:

No.	German version	English translation
2.11	Alles, was nur auf eine einzige Art bestimmt werden kann, muß durch seinen Begriff durchgängig bestimmt werden.	All that can be described in only one way must be completely described by its own concept.
2.21	Das notwendige Wesen muß durchgängig durch seinen Begriff bestimmt werden.	The necessary being must be completely described by its own concept.
2.31	Jeder Begriff, der ein Ding durchgängig *a priori* bestimmt, ist jener des *entis realissimi*.	Every concept that describes a thing *a priori* is that of an *ens realissimum*.
2.32	Ein Ding muß durch seinen Begriff genau dann durchgängig bestimmt werden, wenn es nur dadurch gedacht werden kann.	A thing must be thus precisely and completely described by its own concept if it can be thought of only in this way.
2.41	Alles, was so geartet ist, daß der Begriff des allrealsten Wesens der einzige ist, wodurch es gedacht werden kann, existiert notwendig.	Everything that is of such a nature that the concept of the most real being is the only one through which it can be thought necessarily exists.
2.42	Jedes notwendige Wesen ist ein allerhöchstes Wesen.	Every necessary being is a highest being.

Reconstruction:

Premises:

10. $\bigwedge_x \{Eb(x) \rightarrow \bigwedge_y \bigwedge_z [BS(x, y) \wedge BS(x, z) \rightarrow y = z]\}$

For every x, if x is described in only one way, then for every y and z, if x is described by concepts y and z, then y and z are identical.

11. $\bigwedge_x [N(x) \rightarrow Eb(x)]$

Every necessary being is described in only one way.

12. (M) $\underset{x}{\forall}\underset{y}{\forall}\underset{z}{\forall}\{[BS(x,y) \wedge BS(x,z) \rightarrow y = z] \rightarrow [BS(x,y) \rightarrow Er(y)]\}$

For every x, y, and z, if x is described by concepts y and z, then y and z are identical, then if x is described by concept y, then y is an entis realissimi concept.

(B) $\underset{x}{\forall}\underset{y}{\forall}\{\underset{z}{\forall}[BS(x,y) \wedge BS(x,z) \rightarrow y = z] \rightarrow [BS(x,y) \rightarrow Ber(y)]\}$

For every x and y, if for every z, if x is described by concepts y and z, then y and z are identical, then if x is described by concept y, then y is an entis realissimi concept.

13. (M) $\underset{x}{\forall}\underset{y}{\forall}\{[BS(x,y) \rightarrow Er(y)] \rightarrow En(x)\}$

For every x and y, if x is described by concept y, then y is an entis realissimi concept, then x exists out of necessity.

(B) $\underset{x}{\forall}\{\underset{y}{\forall}[BS(x,y) \rightarrow Ber(y)] \rightarrow En!(x)\}$

For every x, if for every y, if x is described by concept y, then y is an entis realissimi concept, then x exists out of necessity.

14. $\underset{x}{\forall}[H(x) \rightarrow N(x)]$

The highest being is a necessary being.

Proof:

15.		$\underset{x}{\forall}\{N(x) \rightarrow \underset{y}{\forall}\underset{z}{\forall}[BS(x,y) \wedge BS(x,z) \rightarrow y = z]\}$	10, 11, Barbara
16.	(M)	$\underset{x}{\forall}\underset{y}{\forall}\{[N(x) \rightarrow BS(x,y)] \rightarrow Er(y)\}$	12, 15, Barbara
	(B)	$\underset{x}{\forall}\underset{y}{\forall}\{N(x) \rightarrow [BS(x,y) \rightarrow Ber(y)]\}$	12, 15, Barbara
17.	(M)	$\underset{x}{\forall}[N(x) \rightarrow En(x)]$	13, 16, Barbara
	(B)	$\underset{x}{\forall}[N(x) \rightarrow En!(x)]$	13, 16, Barbara
18.	(M)	$\underset{x}{\forall}[H(x) \rightarrow En(x)]$	14, 17, Barbara
	(B)	$\underset{x}{\forall}[H(x) \rightarrow En!(x)]$	14, 17, Barbara

The reconstruction presented above is correct in terms of the rules of logical consequence. However, the use of certain premises is disputable. In Bocheński's

views, sentences 10 and 12 from the reconstruction (2.1 and 2.3 in the text) raise serious doubts. Premise 11 is not obvious either. The editors of (B) introduced amendments in lines 12 and 13. Their goal was not only to modify the controversial premises, but to also modify their formalizations.[19] That reasoning contains premises which are not obvious even on the grounds of Kant's philosophy. Apart from that, the editors of (B) notice the consistency between premises 10–14 and the respective fragments of Kant's text: 2.11, 2.1, 2.3, 2.4, 2.42, and the fact that probably Kant's argumentation ends with sentence 2.4, while 2.5 is only an additional remark. In such a case, the reconstruction presented by Bocheński would be only one of a few possible reconstructions.

The subject of further criticism conducted by Kant is the following sentence, which is absent from his report on the criticized argument:

No.	German version	English translation
2.9	Jedes schlechthin nothwendige Wesen ist zugleich das allerrealste Wesen.	The absolutely necessary being is also the most real being.

As Bocheński reasonably suggests, this is probably sentence 17 from the presented reconstruction. Kant states that he reverses the sentence and formulates the following disquisition:[20]

No.	German version	English translation
2.10	Einige allerrealste Wesen sind zugleich schlechthin nothwendige Wesen.	Some most real beings are at the same time absolutely necessary beings.
2.11	Nun ist aber ein *ens realissimum* von einem anderen in keinem Stücke unterschieden,	But now one *ens realissimum* does not differ the least bit from another,
2.12	und was also von *einigen* unter diesem Begriffe enthaltenen gilt, das gilt auch von allen.	and thus what holds of some beings contained under this concept holds also of all.
2.13	Mithin […] ein jedes allerrealste Wesen ist ein nothwendiges Wesen.	Hence […] every most real being is a necessary being.

[19] Note that, for instance, expression 12 from version (M) is equivalent to the following expression in the first-order predicate calculus with identity: $\forall_x \forall_y [BS(x, y) \rightarrow Er(y)]$.

[20] The English text is quoted from: I. Kant, *Critique of Pure Reason*, op. cit., p. 572.

2.14	Weil nun dieser Satz bloß aus seinen Begriffen *a priori* bestimmt ist, so muß der bloße Begriff des realsten Wesens auch die absolute Nothwendigkeit desselben bei sich fuhren;	Now, because this proposition is determined merely from its concepts *a priori*, the mere concept of the most real being must also carry with it the absolute necessity of this being
2.15	welches eben der ontologische Beweis behauptete	– which is just what the ontological proof asserts
2.16	und der kosmologische nicht anerkennen wollte,	and the cosmological proof does not want to recognize,
2.17	gleichwohl aber seinen Schlüssen, obzwar versteckter Weise, unterlegte.	despite the fact that it underlies its inferences, though in a covert way.

Here we are dealing with two types of reasoning, which Bocheński formalizes in the following way:

Reconstruction:

Premises:

19. (M) $\underset{x}{\exists}\underset{\varphi}{\exists}\{[Rm(x) \wedge \varphi(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow \varphi(y)]\}$[21]

For a certain property and a certain x, if x has that property and is the most real being, then every most real being has that property.

(B) $\underset{\Phi}{\forall}\{\underset{x}{\exists}[Rm(x) \wedge \Phi(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow \Phi(y)]\}$

For every property, if a certain most real being has that property, then every most real being already has that property.

20. (M) $\underset{x}{\exists}[Rm(x) \wedge En(x)]$[22]

(B) $\underset{x}{\exists}[Rm(x) \wedge En!(x)]$

A certain most real being exists out of necessity.

[21] In premise 19 (in both versions), Bocheński uses predicate variables. However, that fact is not mentioned directly here.

[22] In the chapter of *Gottes Dasein und Wesen* we are interested in, in version (M), Bocheński does not explain the symbol "En". For that reason, we understand it in the same way as symbol "En!" in (B).

Proof:

21.	(M)	$\underset{x}{\forall}\{[Rm(x) \wedge En(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow En(y)]\}$[23]	19, En/φ
	(B)	$\underset{x}{\exists}\{[Rm(x) \wedge En!(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow En!(y)]\}$	19, $En!/\Phi$
22.	(M)	$\underset{y}{\forall}[Rm(y) \rightarrow En(y)]$	21, g[24]
	(B)	$\underset{y}{\forall}[Rm(y) \rightarrow En!(y)]$	20, 21, g

The editors of (B) emphasize the consistency between premises 19 and 20, and sentences 2.12 and 2.10 from Kant's text. Premise 19 in version (B) is much stronger than the one from version (M), and it seems that it expresses Kant's thought more adequately. Line 21 in version (M) is equivalent to $\underset{x}{\exists}[Rm(x) \wedge En(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow En(y)]\}$ in the first-order predicate calculus with identity, while in version (B) it is equivalent to $\underset{x}{\forall}[Rm(x) \wedge En(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow En(y)]$. For that reason, in the way shown in the formalization we can obtain line 21 in version (M), but not in version (B). Thus, line 21 should be left in the form it has been given in (M).

Premises:

23.	(M)	$W(9) \rightarrow W(18)$[25]	Proof of sentence 18
	(B)	$W(2.9) \rightarrow W(18)$	Proof of sentence 18
		If sentence 2.9 is true, then also sentence 18 is true.	
24.	(M)	$G(k) \rightarrow W(9)$	Def.
	(B)	$G(k) \rightarrow W(2.9)$	Proof of sentence 18
		If the cosmological argument is valid, then sentence 2.9 is true.	

[23] In the original text it reads $\underset{x}{\forall}\{[Rm(y) \wedge En(x)] \rightarrow \underset{y}{\forall}[Rm(y) \rightarrow En(y)]\}$, which we consider to be an obvious typographic error.

[24] The logical commentary should have the same form here as it has in the book version.

[25] The numbering of lines in (M) and (B) omits 8 and 9. Here and in the subsequent lines what is meant is probably not line 9 from the formalization, but sentence 2.9 (as it is in the book version). In this fragment of the analysis, numbering from Kant's text is mixed with the numbering of lines from the formalizations.

25.	(M)	$W(18) \rightarrow G(o)$	Def.
	(B)	$W(18) \rightarrow G(o)$	Proof of sentence 18
		If sentence 18 is true, then the ontological argument is valid.	
26.		$\sim G(o)$	Proven by Kant (and Aquinas)
		The ontological argument is not valid.	

Proof:

27.	$G(k) \rightarrow W(18)$	23, 24, n
	If the cosmological argument is valid, then sentence 18 is true.	
28.	$G(k) \rightarrow G(o)$	25, 27, n
	If the cosmological argument is valid, then the ontological argument is valid.	
29.	$\sim G(k)$	28, 26, o
	The cosmological argument is not valid.	

Bocheński notices that in light of the truth of premises 24, 25, and 26, the truth of conclusion 29 depends only on the truth of premise 23. Since, in his opinion, it has already been proven that the proof of that premise is not valid, the reduction of the cosmological argument to the ontological argument presented by Kant is incorrect. The comparison of the manuscript with the book shows that the description of premises 24 and 25 is changed. However, finding them in the proof of line 18 is not the only problematic issue – so is treating them as kinds of definitions or sentences resulting directly from definitions. Another controversy is caused by the way of formalization presented by Bocheński, because of the fact that language is mixed here with metalanguage. Additionally, the concept of validity also requires further specifications. Moreover, that concept is not used by Kant in his text. Bocheński realizes that the analyzed fragment of Kant's argumentation is of key importance, but it seems that he conducts the formalization in the least precise way in comparison to the other arguments.

A Criticism of "the Nest of Dialectical Presumptions"

Kant also criticizes the argument of the existence of a necessary being (sentences 1–9) and, to be more precise, the premises assumed in that argument. He does it not when reporting on the cosmological argument, but in the fragment concerning "the nest of dialectical presumptions":[26]

No.	German version	English translation
3.1	Der transscendentale Grundsatz, vom Zufälligen auf eine Ursache zu schließen.	The transcendental principle of inferring from the contingent to a cause.
3.2	Der Grundsatz, von der Unmöglichkeit einer unendlichen Reihe über einander gegebener Ursachen in der Sinnenwelt auf eine erste Ursache zu schließen.	The inference from the impossibility of an infinite series of causes given one upon another to a first cause.
3.3	Die falsche Selbstbefriedigung der Vernunft in Ansehung der Vollendung dieser Reihe.	The false selfsatisfaction reason finds in regard to the completion of this series.
3.4	Die Verwechselung der logischen Möglichkeit eines Begriffs von aller vereinigten Realität (ohne inneren Widerspruch) mit der transscendentalen, welche ein Principium der Thunlichkeit einer solchen Synthesis bedarf.	The confusion of the logical possibility of a concept of all reality united (without internal contradiction) with its transcendental possibility, which requires a principle of the feasibility of such a synthesis.

When analyzing the objections listed above, Bocheński shows that none of them is sufficiently justified because:

3.1. Kant fails to explain why the law of causality should not be used outside empirical experience. This type of interpretation of Kantianism is extremely rare nowadays. In contemporary science, the law of causality is used outside sensual experience, for instance, with reference to the so-called theoretical propositions.

3.2. There are no reasons to reject the possibility of the existence of the first cause in the infinite sequence of causes.

3.3. It is necessary to distinguish between the truth of a given sentence and the fact we accept it for the sake of our pleasure. Kant fails to explain why such a situation should apply to the cosmological argument.

3.4. In this argument, however, nothing is said about the logical possibility of existence. Probably Kant means the difference between proving that

[26] The English text is quoted from: I. Kant, *Critique of Pure Reason*, op. cit., pp. 572–573.

something exists and the actual existence of that thing. Unfortunately, in his view, it contradicts the practice of human reasoning, including scientific reasoning.

Bocheński's Conclusions

These analyses led Bocheński to the formulation of the following conclusions:

1. The proof of God's existence presented and criticized by Kant is not identical with any one proof provided by Aquinas, as it is a combination of the second way and the third way.
2. Causality in Kant's text refers only to non-necessary objects.
3. Kant thinks that the sequence of causes should have the first element in order to be complete. In Bocheński's view, this idea is absent from Aquinas's thought.
4. The reconstruction of the second way, conducted by Bocheński earlier, shows that reducing it to the ontological argument is not necessary.
5. None of the arguments proving that the cosmological argument is not valid, which are discussed by Bocheński, is in his view correct.
6. These arguments can be reconstructed in such a way that they can preserve their formal correctness. Unfortunately, it is not like that with their material correctness.
7. The degree of accuracy in the case of Kant's text is lower than in the case of Aquinas's text, and the schemas of reasoning are trivially simplistic.
8. Kant fails to prove the incorrectness of Aquinas's arguments in the studied fragment. Such a claim is unjustified.
9. This fact has often been overlooked due to Kant's complex style, unfamiliar terminology, and numerous assumptions.

The Significance of Bocheński's Analyses

Although the text concerning Kant and his criticism of the cosmological argument constitutes an appendix to the key analyses of the first questions from the *Summa theologiae* presented in *Gottes Dasein und Wesen*, it is important not only

for historical reasons of showing Bocheński's return to the programme of the Cracow Circle. Looking at the analyzed texts, we can notice that:

1. The available versions of Bocheński's discussed analyses (manuscript/book) do not differ much from each other when it comes to certain formal contents. Apart from that, the book version contains some remarks formulated by its editors. These include, among others, their own proposal of modifying the formalization conducted by Bocheński (footnotes 57 and 62).
2. Bocheński's analyses are an attempt to reconstruct the structure of the argument in the analyzed text. In fact, the chosen text is not easy to analyze as it contains repetitions and the premises used are sometimes implicit. As Bocheński shows, some of them are unjustified or insufficiently justified, even if we base our disquisition on theses that are accepted in Kant's philosophical system, understood in a broad sense. Bocheński fails to comment on the type of premises 1–3, 10–14, and 19–20, and does not say whether they are ontological or empirical, as he used to do in his analyses of the *Summa theologiae*. He also fails to analyze them in detail and classify them in the way he did with reference to Aquinas's text. Bocheński tries to correct errors related to logical consequence at various stages of reasoning on his own. Apart from that, not only does he indicate those premises from Kant's argumentation that are difficult to accept, but he also tries to respond to his objections raised in the fragment concerning "the nest of dialectical presumptions," where Kant directly attacks certain premises from the cosmological argument.
3. Bocheński always starts his analyses with the identification of specific fragments and sentences in the original text. Because of the above-mentioned numerous repetitions, it was problematic to determine which fragments from Kant's text should be formalized. In his analyses, Bocheński paraphrased in formal language the text written in natural language. The advantages of that procedure can be seen, for instance, in the presentation of the argument structure of the text. The obvious direction of those preliminary studies may be the development of a formalized Kantian theory of the necessary being, or the one called *ens realissimum* in this case.
4. The calculus used by Bocheński in his analyses is second-order logic with identity. What is typical of him is the fact that he identifies the rules of

reasoning that are actually used in argumentation. Thanks to this, having finished the analysis, we can determine the language and logical calculus needed for our formalization.

5. A very controversial issue is Bocheński's application of the same language as the object language and its metalanguage (in the fragments concerning the relationships between the cosmological argument and the ontological argument). The use of such predicates as: *G(x)* =: *x ist gültig (x is valid)*, *W(x)* =: *x ist wahr (x is true)*, and constants: *k* =: *kosmologischen Gottesbeweis (the cosmological argument)*, *o* =: *ontologischen Gottesbeweis (the ontological argument*) is at least questionable here.
6. The fragment of Bocheński's formalization that concerns the reduction of the cosmological argument to the ontological argument is controversial also because Kant does not use there the concepts of the validity of an argument or the truth of a sentence. Although that fragment is in fact inconclusive in its interpretation and unconvincing,[27] the real problem results from the fact that without making any reference to the idea of *ens realissimum*, and, consequently, to its existence (as stated in the ontological argument), it is impossible to prove the existence of the necessary being, whose idea, in Kant's view, is postulated by the cosmological argument. Obviously, Kant often writes about reducing the cosmological argument to the ontological argument, but it seems that he means reduction understood in the sense described above.
7. In a number of places, Bocheński makes references to Aquinas. However, this does not seem to be necessary, since in the report on the cosmological argument Kant mentions only Leibniz. The comparison with Aquinas made by Bocheński is inspired by his earlier analyses of the *quinque viae*. Since in Bocheński's view only the second way is valid (its premises are acceptable on the grounds of Aquinas's philosophy, and the argumentation is free from logical errors), he juxtaposes it with the most popular criticism of the cosmological type of arguments. The result of that polemic depends on the acceptance of premises (and rules of reasoning) used in the arguments, and, being dependent on them, it has a local character and is limited to certain philosophical systems. The way schemas of reasoning are

[27] Cf. F. Copleston, *History of Philosophy*, Vol. 6, New York, NY 1993, pp. 297–299.

introduced in the discussed text proves that it was added to the analyses of Aquinas's *Summa theologiae* much later.

8. The results of the analyses conducted by Bocheński confirm what he had written about Kant in his earlier texts. It concerns both his knowledge of logic and his criticism of the cosmological argument. In his *Wspomnienia* [Memoirs], Bocheński writes that "for instance, in Kant's case, it [ignorance of logic] reaches improbable dimensions," and "I used to tell my students that those allegedly great philosophers, including Descartes, Kant, or Hegel, would have failed the first end-of-semester exams if they had been examined by stoics, scholastics, or us – mathematical logicians," and that "the history of logic was unlucky. Immanuel Kant, the most influential philosopher of the modern era, said that logic, unlike other sciences, had never had any history. In Kant's view, Aristotle created it out of nothing, and everything written later was worthless as it destroyed Aristotle's achievements."[28] Thus, comparing Aquinas's artistry in argumentation with Kant is unfavourable for the latter, although the results of Bocheński's analyses are not as adverse as the claims included in the above quotations. When it comes to Kant's criticism of the cosmological argument, Bocheński writes in *Zarys historii filozofii* [An Outline of the History of Philosophy] that "the cosmological argument (based on causality) is also invalid, unless we accept the ontological argument (that thesis is not proven by Kant)."[29]
9. Bocheński's work concerning Kant's criticism of the cosmological argument can be a good reference point for further formal analyses of that argumentation. Moreover, they could be supplemented with analyses of the criticism of the other two types of arguments distinguished by Kant: ontological and physicotheological, as well as with analyses of the division into those three types of arguments. A valuable complementation of the analyses of the cosmological argument would be a logical analysis of Kant's fourth antinomy of pure reason concerning the existence of a necessary being. Apart from possibly improving the analysis presented in this paper, it could provide material for further research.

[28] J.M. Bocheński, *Wspomnienia*, op. cit., pp. 313, 320, 319.
[29] J.M. Bocheński, *Zarys historii filozofii*, Kraków 1993, p. 187.

Despite their numerous drawbacks, not only of formal nature, Bocheński's analyses presented in this paper deserve popularization, both among converted Kantists, such as Bocheński himself, and those who are still inspired by the philosopher from Königsberg.

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Summary

Józef Maria Bocheński is widely known as a promoter of the application of logic to theology and the philosophy of God. His analysis of St Thomas Aquinas's *quinque viae* has become a traditional benchmark for numerous formal analyses of the arguments for the existence of God. Thus, we can say that he was a precursor of formal natural theology, which nowadays is undergoing dynamic developments. Bocheński used formal methods to analyze not only arguments for the existence of God, but also their counterarguments. Conducting those two types of analyses is postulated in his programme of studies on God. In this paper, I will discuss Bocheński's only available case of the second type of analysis mentioned above, in which he considers Immanuel Kant's objections to the cosmological argument.

Key words: J.M. Bocheński, I. Kant, cosmological argument, logic, formal natural theology

Edukacja Filozoficzna
ISSN 0860-3839
DOI: 10.14394/edufil.2020.0017
ORCID: 0000-0001-9539-7482

J.M. Bocheński's Understanding of the World and Logical-Algebraic Structures

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Introduction

Questions about reality, its limits, nature and structure are still the most fundamental and the most difficult questions put to humankind and to philosophers by humanity. Each attempt at rationalizing reality or deepening its structure encounters many obstacles. Endeavours to define it in detail depend on the point of viewing it, that is, on objective, subjective or ontological, as well as existential, perspectives, etc. This work will focus on Józef Maria Bocheński's inclination towards seeing the world and its logical structure from the point of view of ontology.

In section 2, we shall discuss the perception of the world deriving from Bocheński, while in the third section – issues of its logical structure will be dealt with. In section 4, we will present a formal framework of the structure of the world.

The World as an Object of Ontology

Let us start by determining how Bocheński defines ontology. It appears that in his *Autoprezentacja* [Self-Presentation] he defines himself as an Aristotelian and acknowledges the *prote filosofia* of ontology, admitting that it is to him "the most

abstract theory of the object at all."[1] He declares, at the same time, that ontology as a general theory of the object (being) is for him nothing else but formal logic.[2] Both of these disciplines have a common object of studies, although the methods used by them do differ. Ontology, in the given framework, is not only an ontology of real objects, but also that of ideal ones, in a similar way to logic in its contemporary form. If the world is an ontological object, then it is justifiable to ask how it is perceived by Bocheński.

First of all, we should state that the word "world" is ambiguous and can be interpreted in a variety of ways. The manner in which Bocheński perceives objects and the world as an ontological object (being) can be demonstrated using the following selection of his opinions (in compliance with his ontological terminology included in *The Methods of Contemporary Thought*):

> The world is made up of things (elements, substances), such as mountains, plants, men, etc., which are characterized by various properties – e.g. colors, shapes, dispositions, etc. – and linked one with another by a variety of relations. The general philosophical name for everything which is or can be is "being" (*Seiendes*); even such things as properties and relationships will thus be called "beings". It is possible to distinguish two aspects in every being: what it is – its nature, its "whatness", its essence – and the aspect which consists in the fact that the being is, its *Dasein*, its existence.[3]

In another place in his self-presentation, Bocheński writes that the world consists primitively of units (substances) determined by certain properties and connected by defined, often also real, relations.[4] Therefore, the world in Bocheński's understanding is a whole composed of all beings with certain properties and connected by relationships. Does this mean that Bocheński perceives the world solely as a world of real objects?

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1 J.M. Bocheński, *Autoprezentacja*, trans. J. Garewicz, in: J.M. Bocheński, *Logika i filozofia. Wybór pism*, ed. J. Parys, Warszawa 1993, pp. XXVII, XXVIII [Ger. orig.: *Sellbstdarstellung*, in: *Philosophie in Sellbstdarstellungen I*, ed. L.J. Pongratz, Hamburg 1975, pp. 1–36].

2 Bocheński writes about the mutual relation between logic and ontology from the perspective of Western history in J.M. Bocheński, *Logika i ontologia*, trans. D. Gabler, in: J.M. Bocheński, *Logika i filozofia. Wybór pism*, op. cit., pp. 106–132 [Eng. orig.: *Logic and Ontology*, "Philosophy East and West" 1974, Vol. 24, No. 3, pp. 275–292].

3 J.M. Bocheński, *The Methods of Contemporary Thought*, trans. P. Caws, Dordrecht 1965, p. 3.

4 J.M. Bocheński, *Autoprezentacja*, op. cit., p. XXVIII.

According to him, objects are not only things, but also each object understood very broadly, that is, as something about which a statement can be made. Ontological objects are things and also their properties, as well as the relations between them. Such objects are states of affairs as well, since Bocheński writes that

> if a being is modified in some way – e.g. if a thing is red, or a geometrical figure has twice the area of another – we are confronted with a state of affairs.[5]

And a little further:

> States of affairs are not independent of one another. On the contrary, it often happens that if one state of affairs is the case, then another is also the case. The world may be thought of as an interrelated pattern of states of affairs. Indeed it is itself a colossal and extremely complicated state of affairs, in which everything that is or can be is connected with everything else in an endless network of relationships.[6]

The world perceived ontologically, in accordance with Bocheński's vision, is – at the same time – a complete whole of harmoniously ordered objects, which – apart from all the bodily objects – are composed of their properties and the relations established between them.

It follows from Bocheński's argumentation that the world to him is not exclusively a material, real world, though. As a rationalist he comprehends it not only as a palpable reality, but also as an ideal and potential one, possessing a logical structure anyway. The world is to him, like things, features and relationships, an ontological object, a being, an object of studies of ontology as a universal theory of beings.

According to Bocheński, objects are also ideal beings. He defines himself as a Platonian when he writes the following:

> It is my Platonism that separates me from my positivist friends, the conviction that there exists an ideal object and not only real ones and (sharing the belief with Whitehead) that there is no explanation of what is real without reference to the ideal being. […] Habitually, we reject the ideal object verbally only to have it smuggled in shortly afterwards.

[5] J.M. Bocheński, *The Methods*…, op. cit., p. 2.
[6] Ibid., p. 3.

> […] From the ontological viewpoint, my Platonism asserts that there exist also ideal things, content, etc. It is true that I do not think that the things are flying somewhere around "in the world" and I am rather inclined to accept that they arise only thanks to people. But man recognizes such things. […] they are not his thoughts […] but the content of thoughts. They are given objectively.[7]

Advocating the Aristotelian-Kantian perspective, Bocheński accepts that "whatever is ideal, is a form of whatever is real and can be extracted from there in some way by our intellect and updated."[8] The ontology of the object has a few varieties in Bocheński's view: ontology of the real object, ontology of the object and property, ontology of conditions (relational and functional), ontology of the ideal object and the like.

At the same time, Bocheński accepts the primacy of the real object over the ideal one. Earlier, as a rationalist, he wrote:

> It is believed that we cannot know everything fully. It is very possible that there exist things which are cognizable only externally, isomorphously, as we used to say in logic. Obviously, such a thing is God and there may exist also other ones. My rationalism says, however, that what we are able to know about things, we know it within logic – not outside it – thanks to logical means.[9]

Thus, Bocheński differentiates the cognizable reality from the world as such, perceived as broadly as it is only possible, containing all classes of beings. Such a world includes not only the real world (treated as the set of all the cognizable objects in common experience, the world of temporal beings, beings existing in time). It comprises the world of nature as well (a concrete occupying the whole space at any given time).[10] According to Bocheński, the world is the whole of reality understood as widely as possible, comprising both material and ideal beings. The world to Bocheński is heterogenic and not only homogenic (the material

[7] J.M. Bocheński, *Autoprezentacja*, op. cit., pp. XXV, XXVI. All quotations have been translated by Jacek Jędrzejowski.

[8] Ibid., pp. XXV, XXVI.

[9] Ibid., p. XXII.

[10] See A. Biłat, *The World as an Object of Formal Philosophy*, in: *Contemporary Polish Ontology*, ed. B. Skowron, Berlin–Boston, MA 2020, pp. 87–108. The concrete (the world of nature) is here a collective, mereological set, the largest material whole; it occupies the entire space at any given time, while the real world is here the set of all concretes, i.e., the empirically recognizable objects that occupy a certain place at a certain time. The real world does not include any relations or ideal objects that belong to the whole world.

wholeness of bodily beings, a collective set, homogenous as far as certain of their properties are concerned, for example, temporality, variability, etc.).[11]

Relationships between the concepts of the world as a extensional whole (W), the real world (rW) and the world of nature (nW) in Bocheński's framework can be written as follows:[12]

$$nW \in rW \subset W,$$

where "$\subset$" denotes the set-theoretical proper inclusion and "$\in$"denotes the membership relation.

The Logical Structure of the World

In his *Autoprezentacja*, Bocheński defines himself as a rationalist and, stressing the importance of the intellect, he writes:

> The intellect seen objectively is the same as formal logic. There results thus the following vision of the world: it is a colossal, to the highest degree complicated, mass of things, properties and occurrences. Still, impenetrable though it appears at the beginning, it does possess a completely defined (static and dynamic) *structure*. Indeed, the "structure" is another word that means "a network of relations." Formal logic, on the other hand, is nothing more than the most general theory of relations. This means that the world has logical structure. It even seems to me that the expression "a non-logical structure" is a contradiction.[13]

Therefore, it may be assumed that in Bocheński's framework the world has a relational structure, and it contains a network of relationships possessing formal properties that enable their description. These formal properties are the domain of the theory of relations, which is the most general section of formal logic.

The formal description of the structure of the world, its organization, does not include the description of individual properties of beings or the description of specific relations between them. The description of the formal properties of

[11] See J. Herbut, *Leksykon filozofii klasycznej*, Lublin 1997, p. 501.
[12] Cf. A. Biłat, *The World*…, op. cit., p. 96.
[13] See J.M. Bocheński, *Autoprezentacja*, op. cit., pp. XX, XXII.

relations, which are components of the world's structure and types of these relations, is established in the theory of relations as a logical theory. It is this theory that serves to analyze reality and to identify the world's structure as a relational structure.

It can be supposed that Bocheński knew the works of Rudolf Carnap, who precisely defined what he understood by formal properties of a relation:

> By formal properties of a relation, we mean those that can be formulated without reference to the meaning of the relation and the type of objects between which it holds. They are the subject of the theory of relations. The formal properties of relations can be defined exclusively with the aid of logistic symbols, i.e., ultimately with the aid of the few fundamental symbols which form the basis of logistics (symbolic logic). (Thus these symbols do not specifically belong to the theory of relations, but form the basis for the entire system of logic–propositional logic, the theory of propositional functions (concepts), the theory of classes, and the theory of relations.)[14]

Bocheński perceives himself as a rationalist. "My rationalism," he writes, "consists first of all in that I have always treated reality as cosmos, not as chaos."[15] To him the world is a logical cosmos. In opposition to chaos it has an ordered structure, it is an ordered whole.

Thus, the structure of reality as a network of relations is a whole, a system of logically distributed, mapped-out objects connected by relations. Bocheński believes that whatever stands against logic cannot exist in the world.

Bocheński realizes that such a statement raises a great deal of questions about people, and so he answers it in the following way:

> How can we know that the world is a logical cosmos? The answer to this question seems simple: The whole of human experience and – first of all – the experience of the natural sciences, assumes this thesis and it has always turned out that the thesis is confirmed. In the face of this fact it seems to me simply unreasonable to doubt the *logical structure of the world*.[16]

[14] R. Carnap, *The Logical Structure of the World*, trans. R.A. George, Berkeley–Los Angeles, CA 1969, Part Two, Chap. A, Sec. 11, p. 21 [Ger. orig.: *Der Logische Aufbau der Welt*, Wien 1928].

[15] J.M. Bocheński, *Autoprezentacja*, op. cit., p. XX.

[16] Ibid., p. XXI (emphasis added).

Again, in his *Autoprezentacja*, Bocheński promotes quietened theory-cognitive *optimism*, which consists in

> some dose of trust in the human intellect, […] in accepting the fact that although it is with difficulty and in an incomplete manner, we still can recognize certain states of things in the way that they are, and this without expressing towards them any fear or disgust, but through a common experience and reasoning.[17]

Further in his work, Bocheński states that for him, as a rationalist,

> [p]aradoxically, the anti-sceptic attitude arises from the belief that the world is extraordinarily complicated and that we can recognize very little of it. The reality is not exclusively the same as we perceive it to be. It does not limit itself to human experience. It is most likely that we do not know many things, many are known only superficially, yet we do know *something very precisely and for sure*. The whole human experience speaks for rationalism, and there is nothing to speak against it.[18]

Bocheński advocates the rationalization of the world as one possessing the logical structure that can be regarded as its foundation, which determines it as a whole of its harmoniously ordered elements, objects, beings.[19] These elements, beings or objects are not only those really existing, but also ideal ones.

The world *W*, according to Bocheński, has a rational, logical and relational structure, which means that its foundation, a logically ordered system of coordinated elements, is a set of hierarchically ordered beings. It is a certain whole conditioned by a stable logical ordering of its components and linked by means of relations. Its components, elements are things and states of things as well as ideal individuals (zero-argument relations), possessing certain properties or features (one-argument relations) and connected by many-argument relations. This order of the components, elements of the world is the source of the *cosmic order*, harmonies which are revealed through the laws of nature.

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[17] Ibid., pp. XXII–XXIII.

[18] Ibid., p. XXIII (emphasis added).

[19] Cf. B.K. Krzych, *Struktura rzeczywistości*: *jednowielorakość?*, "Amor Fati" 2017, Vol. 2(8), pp. 301–319.

The Formal Structure of the World

The considerations of the previous parts of this work can be given a formal, logical shape. In this respect, something is a structure when it constitutes a formally determinable relational structure.

The relational *structure*[20] of the world W (reality) is an ordered triple:

$$\mathrm{W} = < W, \{R_i\}_{i \in U}, \{O_j\}_{j \in S} >,$$

where the world W (called domain) is a non-empty set of all things, states of things and ideal individuals, U and S are subsets of the set N of all natural numbers, $\{R_i\}_{i \in U}$ is an indexed set of all unary relations (properties, features) on W and many-ary relations on W, while $\{O_j\}_{j \in S}$ is an indexed set of operations on W. If it is empty, the structure W is a pure relational structure, and if the set of relations is empty, this structure is an algebra.

As we have already mentioned, the world W, according to Bocheński, is a logical cosmos. As such, standing in opposition to chaos, it is organized by some hidden, internal natural *order*,[21] by a certain solid relation ordering the nucleus of the world's structure and all of its elements.[22] The leading philosophers and ontologists of our "Western" cultural circle basically agree that that cosmos – in the language of philosophy – is a world conceived as an internally ordered whole, in contrast to chaos. Reconstructing Bocheński's conception of the logical structure of the world in a formal way, it needs observing that the relational structure W of the world W assumes that the components of its domain W are ordered. We denote the ordering relation on W by $\leq$; it cannot be a relation belonging to the world W.[23] Then, in Bocheński's framework, the logical structure of the world W is represented by the following, slightly enriched system:

$$\mathrm{W}' = < (W, \leq), \{R_i\}_{i \in U}, \{O_j\}_{j \in S} >,$$

[20] See W.A. Pogorzelski, *Notions and Theorems of Elementary Formal Logic*, Białystok 1994, p. 386.

[21] See A. Grzegorczyk, *W poszukiwaniu ukrytego sensu. Myśli i szkice filozoficzne*, Lublin 2018, Part 3.1: *Ukryty porządek świata*, pp. 211–219.

[22] It needs noting that among the relations R_i in the structure W there can be found many relations which order spatially, temporally, causatively or in any other manner. They are not relations which order the whole universum W, though.

[23] It can be understood as a *universal* relation holding between all elements of the whole world W. Relations R_i belong to the world W and they are defined for some classes of elements of the world W.

in which ≤ is a binary ordering relation on *W*, that is, a reflexive, asymmetric and transitive relation on *W*.

To Bocheński, the structure of the world is a network of relations, a configuration of beings which are mutually connected to one another by the relations, suitably logically distributed, mapped out in the substructure (*W*, ≤) of the structure W′. It can be supposed that the structure (*W*, ≤) forms a logical *algebraic lattice*.[24] At the same time, both the temporal world – a dynamic one with the domain of temporal beings (beings existing in time) – and the static world – one with the domain of static beings treated as an extensional whole – have a logical structure. It is disputable whether its structure (*W*, ≤) has the form of a logical-algebraic lattice.

Reconstructing formally the conception of the logical structure of the world on the basis of the views put forward by Bocheński in several of his letters, I embarked on faithfully recreating his ideas or viewpoints on the world and its structure. Some general stances on the world and its extensional whole had already been worked out by European philosophers in antiquity and in the Middle Ages. In the last century, the best-known idea of the conceptual structure that categorized the world from the perspective of ontology and logic was expounded in Carnap's seminal work *Der logische Aufbau der Welt* (1928).[25] Apart from this, an outline of the general conception of the world's structure can be found in works of contemporary philosophers. For example, we find some ontological-logical considerations dealing with the world as a set of things possessing different properties and being connected by relations, as well as forming different sets, in the book *Mała propedeutyka filozofii naukowej* [A Short Introduction to Academic Philosophy] by Andrzej Grzegorczyk.[26] In this author's framework, the world is not only, as Bocheński argued, a world of real beings, observable, but also possesses ideal ones (non-observable, imagined or merely thought about), which have certain properties.[27] In frameworks and descriptions of the logical structure

[24] This supposition is based on the observation that the word "structure" was and still is used in philosophy, logic and algebra with a variety of meanings, specifically just like that of "lattice," meaning an ordered set in which each two elements have *infimum* and *supremum* (see, e.g., S. Krajewski, ed., *Słownik pojęć filozoficznych*, Warszawa 1996, p. 187, and G. Birkhoff, S. Mac Lane, *Przegląd algebry współczesnej*, Warszawa 1966, pp. 375, 376).

[25] R. Carnap, *The Logical Structure of the World*, op. cit.

[26] A. Grzegorczyk, *Mała propedeutyka filozofii naukowej*, Warszawa 1989, Chap. 2: *Ogólna struktura świata*.

[27] Ibid., p. 23.

of the world that are known to me, indeed, it is hardly possible to come across a formal reconstruction of such a structure that would comply with the ideas of Bocheński's conception, which can be "fished out" of his letters.

Man in Bocheński's World

The world as a whole, according to Bocheński, consists of things, states of things and ideal objects. Man occupies an important place in this world. He is not the creator of the world but its co-creator and as such bears responsibility for the world, is obligated to take care of its being and his own in particular. Consequently, he is faced with certain tasks and commitments. The care for the being is made real in his rational, wise acting and behaving, and invests him with a sense of the reasonability of his whole life. The greatness of man consists precisely in that he is capable of creative reacting to his own fate, making sensible choices, shaping his existence and rational participation in the existence of community.

The peculiar greatness of Bocheński consists in the fact that although he performed many different roles[28] ("he was a scholar, soldier, priest, monk, teacher, advisor to governments, political activist, publicist and journalist, pilot, driver, preacher, 'star' of international congresses, organizer and builder […] and acted well beyond each of them"), he was able to describe in a simple manner indications relating to man's struggle with secrets of the world and life. Bocheński writes about man's place in this world, among others, in the *Introduction* to his *Podręcznik mądrości tego świata* [A Coursebook on the Wisdoms of This World], in which we read:

> Man himself and all his creations in the form of nations, cultures, achievements of science, etc., are fragments of no importance in the Universe. The world is in particular a real cemetery of dead civilizations and nations. Everything sinks in nothingness. What is more, our inner lives depend to a great extent on this world, on the laws of nature. […] Man is a tiny, helpless fragment of the world, existing only during a fraction of the cosmic second, yet

[28] See Cz. Porębski, ed., *…Skoro Pan Bóg raczył mi w swojej dobroci dać trochę rozumu, to po to, abym go używał…*, "Znak" 1995, Vol. 481, p. 14.

> this fraction is everything we have at our disposal. How to proceed so as not to waste it – this is taught to us through commandments of wisdom.[29]

Father Bocheński considered the commandment "Proceed in such a way as to live a long and prosperous life" to be the fundamental, the first and the most significant principle of wisdom. Several dozen other principles follow from the above-quoted one, with the inclusion of experimental data.[30] It is in them that Bocheński provides recommendations on how to live in the modern world and shows what the sense of individual beings is.

Acknowledgements

The author wants to thank the reviewers of the paper for their opinions, comments and suggestions, which allowed her to make certain statements more precise or complete.

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[29] J.M. Bocheński, *Podręcznik mądrości tego świata*, Kraków 1994, pp. 2, 3.

[30] Bocheński lived to be 93 years old and wrote the coursebook when he was over 90.

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Summary

This paper will focus on J.M. Bocheński's inclination towards seeing the world and its logical structure from the point of view of ontology. Accordingly, on the basis of Bocheński's selected utterances we will present and discuss the perception of the world proposed by the scholar, and then we will deal with questions pertaining to the logical structure of the world and examine a formal framework of this structure.

Key words: Bocheński, ontology, being, world, structure of the world, algebraic relational structure

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ISSN 0860-3839
DOI: 10.14394/edufil.2020.0018
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Formal Theodicy: Religious Determinism and the Logical Problem of Evil

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Introduction

The use of logical tools in the investigation of religious discourse can be traced back as far as the ancient times, but was particularly common in specific periods of some religious traditions. Several authors, for example Thomas Aquinas, endeavoured to use in their texts the rigour and precision that the available logical tools provided. As heirs of such a tradition, Józef Maria Bocheński, Jan Salamucha, Jan Drewnowski, and Bolesław Sobociński formed the so-called Cracow Circle (1934–1944),[1] which aimed at employing the most current methods of mathematical logic in matters of the philosophy of religion and theology. Their ideas and achievements were so important that, according to Roger Pouivet, the philosophers of religion of the Cracow Circle are "the principal precursors of what is now called the analytic philosophy of religion."[2] According to Bocheński, the Cracow Circle postulated that:

1) the language of philosophers and theologians should exhibit the same standard of clarity and precision as the language of science;

[1] See R. Murawski, *Cracow Circle and Its Philosophy of Logic and Mathematics*, "Axiomathes" 2015, Vol. 25, pp. 359–376.

[2] R. Pouivet, *On the Polish Roots of the Analytic Philosophy of Religion*, "European Journal for Philosophy of Religion" 2011, Vol. 3, No. 1, p. 1.

2) in their scholarly practice they should replace scholastic concepts with new notions now applied by logicians, semioticians, and methodologists;
3) they should not shun the occasional use of symbolic language.[3]

In this sense, philosophical and theological doctrines can benefit greatly from employing formalization. This logical approach introduces exactness, clarity, and precision in concepts and arguments and makes it possible to eliminate several kinds of ambiguities.

Although there have been some initiatives in the second half of the past century involving this way of doing philosophy of religion and theology and an increased interest in the study of the relationship between logic and religion, as well as in the use of logical tools by analytic philosophers of religion, the goals outlined by the Cracow Circle are far from achieved. The construction of formalized systems in the philosophy of religion is an approach not as common as it could be. The belief that such an enterprise has several advantages in solving philosophical problems has propelled the authors of this paper into the investigation of a famous issue in the philosophy of religion. The approach used here can be considered a case of logic of religion, as proposed by Bocheński; particularly, it is a case of logic applied to theodicy.[4] In what follows, we delineate this problem.

1. Systems N1, N2, and the Logical Problem of Evil

The problem of evil is one of the most prominent issues in the history of philosophy. Though many answers have been provided since ancient times, many authors still maintain that it is a challenge to belief in God. One can approach the problem in many ways; however, in the contemporary debate, there are at least two strands of this problem. Some philosophers have argued that the existence of God is very improbable, given the amount and variety of evil in the world; this proposal has been called the *evidential problem of evil*.[5] Others claim that the propositions "God exists" and "evil exists" are mutually inconsistent, and thus theists cannot be rational. John Mackie is one such philosopher; as he says,

[3] J.M. Bocheński, *The Cracow Circle*, in: *The Vienna Circle and the Lvov–Warsaw School*, ed. K. Szaniawski, Dordrecht 1989, pp. 9–18.

[4] On the logic of religion, see Bocheński's seminal work, *The Logic of Religion*, New York, NY 1965.

[5] For a comprehensive overview on this issue, cf. D. Howard-Snyder, ed., *The Evidential Argument from Evil*, Indianapolis, IN 1996.

> [A] more telling criticism can be made by way of the traditional problem of evil. Here it can be shown, not that religious beliefs lack rational support, but that they are positively irrational, that the several parts of the essential theological doctrine are inconsistent with one another.[6]

The affirmation that belief in God is contradictory with the existence of evil is called the *logical problem of evil*, and it is our focus here. One of the most influential responses to Mackie is the free will defence, developed by Alvin Plantinga. He develops his defence in modal metaphysics and semantics of possible worlds, to consider whether God could have created a world with less moral evil – or even no moral evil at all. Plantinga concludes that it is possible that God could not have done this; it is possible that He had a reason to permit the existence of evil in the world, and, thus, there is no contradiction between the existence of God and the existence of evil.[7]

Although the free will defence can still be considered relevant nowadays, it relies on some assumptions that remain a matter of discussion. Just to provide an example, one of its most debatable issues is the property of "transworld depravity" (TWD), a property needed to show that it is possible that God could have had a reason to allow evil in the world. However, as Richard Otte has shown, the original version of TWD is necessarily false, and Plantinga himself recognized it.[8] The current version of the argument relies on the version of TWD given by Otte, but one might wonder whether the argument with Otte's TWD is the definitive version of a free will defence; as there is no way to *demonstrate* its logical possibility, the success of such an account depends on the success of its underlying modal metaphysics, and to the extent that natural language allows concepts to be defined with no ambiguity.

Furthermore, even if the free will defence succeeds, other proposals may provide alternative paths to tackle the question.[9] Meanwhile, we could also conceive

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6 J.L. Mackie, *Evil and Omnipotence*, "Mind" 1955, Vol. 64 (254), p. 200.

7 A. Plantinga, *God, Freedom, and Evil*, Grand Rapids, MI 1977, pp. 7–73.

8 R. Otte, *Transworld Depravity and Unobtainable Worlds*, "Philosophy and Phenomenological Research" 2009, Vol. 78, No. 1, pp. 165–177; A. Plantinga, *Transworld Depravity, Transworld Sanctity, & Uncooperative Essences*, "Philosophy and Phenomenological Research" 2009, Vol. 78, No. 1, pp. 178–191.

9 There are plenty of proposals: both defences (possible answers that invalidate an objection from evil) and theodicies (responses to the question of why God permits evil in the world), and even arguments from evil for the existence of God. See J. McBrayer, D. Howard-Snyder, *The Blackwell Companion to the Problem of Evil*, Oxford 2013, for a comprehensive sample of such approaches.

alternative approaches that have not been explored yet; for instance, instead of just using the semantics of possible worlds and modal metaphysics, why not appeal to *full-blown* formal logic? Formal logic improves precision and clarity of arguments, removes ambiguity, brings new results that could not be attained through natural language, and even preserves truth in a way that natural languages are not able to. Why not refer to such a powerful tool? As the Cracow Circle did, concerning many relevant philosophical and religious questions of their days, we also do believe that logical questions require logical answers, and the logical problem of evil is not different. In what follows, we introduce this proposal.

Edward Nieznański developed two logical systems to deal with the task of a formal theodicy.[10] His aim is, in principle, to provide an answer to the problem of evil, especially in its logical form, but his systems offered a wider framework to consider questions related to religious determinism, the attributes of God, and formal axiology. One of the merits of his systems is that they characterize the logical problem of evil, a question usually stated in terms of contradiction and consistency, as a matter of logical investigation. Much of the contemporary debate on the logical problem of evil is done within a modal metaphysics framework, with no explicit formalization,[11] and Nieznański's approaches have the merit of dealing with the question by developing such formal systems.

Nieznański's systems have philosophical relevance, and, further, his general methodology is very inspiring as an application of formal tools to philosophical problems. However, some logical issues led us to revisit his systems, proposing a few changes. For instance, consider the following list of formulas, the original axioms of his first system.[12] In these formulas, β stands for God, x is a vari-

[10] E. Nieznański, *Aksjomatyczne ujęcie problemu teodycei*, "Roczniki Filozoficzne" 2007, Vol. 55, No. 1, pp. 201–217; E. Nieznański, *Elements of Modal Theodicy*, "Bulletin of the Section of Logic" 2008, Vol. 37, No. 3/4, pp. 253–264.

[11] Plantinga's contribution is not only the most influential answer to the logical problem of evil, but it also determined the trend of dealing with this subject in the context of metaphysics of modality and possible worlds semantics. Among the works that follow Plantinga on dealing with the problem, but in natural language, are D. Howard-Snyder and J. O'Leary-Hawthorne, *Transworld Sanctity and Plantinga's Free Will Defense*, "International Journal for Philosophy of Religion" 1998, Vol. 44, No. 1, pp. 1–21; W.L. Rowe, *In Defense of "The Free Will Defense": Response to Daniel Howard-Snyder and John O'Leary-Hawthorne*, "International Journal for Philosophy of Religion" 1998, Vol. 44, No. 2, pp. 115–120; R. Otte, *Transworld Depravity*…, op. cit.; and A.R. Pruss, *A Counterexample to Plantinga's Free Will Defense*, "Faith and Philosophy" 2012, Vol. 29, No. 4, pp. 400–415.

[12] E. Nieznański, *Aksjomatyczne ujęcie problemu teodycei*, op. cit., pp. 203–211.

able that stands for persons, p and q are variables that stand for situations, W is a symbol for "knows that," C is a symbol that means "wills that," D is a symbol for "permits that," $p \varepsilon d$ means "the situation p is good," and, finally, P stands for "to be the cause of":

A1. $\forall x\ (\exists p\ xWp \wedge \exists q\ xCq)$

A2. $\forall p\ [\beta Cp \rightarrow \beta C(\beta Cp)]$

A3. $\forall p\ [\beta Dp \rightarrow \beta C(\beta Dp)]$

A4. $\forall p\ [\exists x\ \beta C(xCp) \rightarrow \forall x\ \beta D(xDp)]$

A5. $\sim\forall p\ (p \rightarrow p\ \varepsilon\ d)$

A6. $\forall p\ (\beta Pp \leftrightarrow \beta Cp)$

When we examine these axioms, from the point of view of mainstream logic, some questions naturally arise: what is the underlying logical system? What is the precise meaning of the symbol ε? Do the symbols W, C, D and P represent functions, predicates or modal operators? And, finally, how to justify the use of variables in formulas such as those in A5? These questions are surely relevant, but there is not much discussion about the specific logical structure in Nieznański's article. A more robust account is provided in his second system, a modal account,[13] but some of the questions listed above can also be applied to this second approach.

Nevertheless, we maintain that Nieznański's insights are philosophically penetrating,[14] and his general methodology of formalizing concepts is inventive and very inspiring. For this reason, these issues, among others, led us to work on a detailed treatment. Thus, we have proposed two revisited systems, as a revisiting (or remaking) of Nieznański's approaches; the first of these systems is **N1**,

[13] E. Nieznański, *Elements of Modal Theodicy*, op. cit.

[14] It is not possible to summarize all of the relevant insights of Nieznański's works here. We encourage interested readers to acquaint themselves with his works in detail to see how the philosophical concepts that he explores rely upon good intuitions concerning the nature of God, the relation between situations, and the relation between will and other features of agents, such as coherence and responsibility; see E. Nieznański, *Aksjomatyczne ujęcie problemu teodycei*, op. cit.; E. Nieznański, *Elements of Modal Theodicy*, op. cit.

published recently in an article, and the second is **N2**, to be published soon.[15] Of course, the line that distinguishes a "revisiting" from a "remaking" is sometimes obscure, but our proceeding with both systems can be synthesized as follows: first, we re-established the formal language to create one that, according to our vision, is more adequate for accomplishing Nieznański's tasks; we described both **N1** and **N2** as first-order modal systems, with two modal operators: $\mathcal{W}_\theta$ ("God knows") and $\mathcal{C}_\theta$ ("God wills"). In our interpretation, these operators are sufficient for dealing with some of the main subjects that Nieznański is concerned with in his articles. Furthermore, we established the formal language, the rules of inference, and other features. Then, we defined a new set of axiom schemes, many of them inspired by Nieznański's work, but with a new formulation, to finally prove some theorems.

The resulting systems have much of the original basic structure, and many axioms, definitions and theorems remain unchanged – yet some new results are obtained. In particular, both **N1** and **N2** characterize the attributes of God, provide a formal axiology, and give an answer to a form of religious determinism; all of these results are also obtained from their distinct sets of axioms. However, when considered together, one of the outcomes is surprising: the systems are mutually *contradictory*. Let us first consider the following formulas:

(1) $\forall p\ [\beta Dp \rightarrow \beta C(\beta Dp)]$

(2) $\sim\forall p\ [\beta Dp \rightarrow \beta C(\beta Dp)]$[16]

These formulas are axioms of Nieznański's systems: formula (1) belongs to the first one, and (2) belongs to the second. The first-order modal versions of them, with the correspondent standard interpretations, are the following:

(1′) $\forall p\ (\mathcal{D}_\theta \alpha(p) \rightarrow \mathcal{C}_\theta \mathcal{D}_\theta \alpha(p))$

(For all situations, if God permits some state of affairs, then God wills to permit such state of affairs.)

[15] G.B. da Silva, F.M. Bertato, *A First-Order Modal Theodicy: God, Evil, and Religious Determinism*, "South American Journal of Logic" 2019, Vol. 5, No. 1, pp. 49–80; G.B. da Silva, F.M. Bertato, *God, Evil, and Religious Determinism: Another First-Order Modal Theodicy*, forthcoming.

[16] E. Nieznański, *Elements of Modal Theodicy*, op. cit., p. 259. The formula written here in the notation of Nieznański's first system is presented in his second system as axiom A6, which in another notation is given by $\sim\forall p(Dbp \rightarrow CbDbp)$.

(2′) $\neg\forall p(\mathcal{D}_\theta\,\alpha(p) \rightarrow \mathcal{C}_\theta\,\mathcal{D}_\theta\,\alpha(p))$

(Not all situations are such that if God permits some state of affairs, then God wills to permit such state of affairs.)

As is evident, the axioms are explicitly contradictory: (1) contradicts (2), and (1′) contradicts (2′). These contradictions lead us to at least two possible conclusions: either one or both systems are trivial, and thus all of the results could be trivially obtained, or, there is a set of axioms which is sufficient to reach some of the most interesting conclusions for both systems, avoiding contradictions.

But the conclusions obtained are not trivial. Indeed, our research finds that there is a set of axioms which satisfies the demands of a new axiomatic system, similar to **N1** and **N2**. The new system, called **N3**, is based on both previous systems, but has only three axioms (much less than **N1** or **N2**, each one with eleven axioms). We think that these axioms are sufficient to prove the most relevant results of **N1** and **N2**, as well as the results Nieznański aimed at: **N3** proposes an answer to the problem of evil through the refutation of a version of religious determinism, showing that the attributes of God in classical theism, namely, those of omniscience, omnipotence, infallibility, and omnibenevolence, when adequately formalized, are consistent with the existence of evil in the world. On the one hand, these questions are also tackled by Nieznański's systems, but, on the other hand, they are obtained in **N3** with fewer assumptions.

In the following, we present the formal structure of **N3**.

2. N3: A Minimal System

2.1. The Language, Rules, and Axioms of N3

The first-order modal language $\mathfrak{L}_{N3}$ of **N3** is composed of the following symbols as primitives:[17]

(i) Unary predicate symbols: B, P, δ, ξ;

(ii) A constant symbol (a distinguished element): θ;

[17] Concerning first-order modal logic, see G.E. Hughes, M.J. Cresswell, *A New Introduction to Modal Logic*, London 1996; W. Carnielli, C. Pizzi, *Modalities and Multimodalities*, Logic, Epistemology, and the Unity of Science 12, Dordrecht 2008; M. Fitting, R.L. Mendelsohn, *First-Order Modal Logic*, Dordrecht 2012. These works have provided useful guidance in the development of this work.

(iii) Variables for situations: p, q, r, possibly with subscripts;[18]
(iv) The symbols for connectives: $\neg$, $\rightarrow$, $\vee$, $\wedge$, $\leftrightarrow$;
(v) The symbols for operators: $\forall$, $\exists$;
(vi) Two symbols for specific modal operators: $\mathcal{C}_\theta$, $\mathcal{W}_\theta$.[19]

The definition of a well-formed formula (*wff*) is the usual, with the expected extensions. The interdefinibility of connectives and operators is also the usual.

The basic rules of deduction of **N3** are: *Modus Ponens* (MP), *Uniform Substitution* (US), *Rule of Necessitation* (Nec) and *Substitution of Equivalents* (Eq). They are stated below:

(MP) $\phi, \phi \rightarrow \psi \vdash_{N3} \psi$.

(US) The result of uniformly replacing any variable or variables $p_1, \ldots, p_n$ in a theorem by any *wff* $\phi_1, \ldots, \phi_n$, respectively, is itself a theorem.

(Nec) If $\vdash_{N3} \phi$, then $\vdash_{N3} \mathcal{W}_\theta \phi$ and $\vdash_{N3} \mathcal{C}_\theta \phi$.

(Eq) If ϕ is a theorem and ψ differs from ϕ in having some *wff* μ as a subformula in one or more places where ϕ has a *wff* γ as a subformula, then if $\mu \leftrightarrow \gamma$ is a theorem, ψ is also a theorem.[20]

As a *convention*, in **N3**, $\alpha(p)$ stands for any *wff* that involves *only* the variable p, where p is free. Thus, we distinguish in **N3** two types of situations, namely, *basic situations* (of the world) and *situations involving situations* (which are here called *states of affairs*). The variables for situations correspond, therefore, to basic situations. In turn, formulas that contain occurrences of free variables for situations represent states of affairs. We are particularly interested here in a specific type of state of affairs, namely, one in which a certain situation p is the case, represented by $P(p)$. Thus, formulas in the form $\alpha(p)$ with only one free variable, p, are the

[18] Despite considering variables for situations, a variable can assume the value θ. It is not intuitive to say that God is a situation, and therefore we could say that the symbols p, q, and r represent either God or situations. **N3** deals explicitly only with God and the (possible) situations of the world. Thus, in addition to dealing with situations, we can express in **N3** sentences such as "God is good," "God is not contingent," etc.

[19] **N3** is a type of First-Order Epistemic-Boulomaic Modal Logic. $\mathcal{W}_\theta$ is an epistemic operator that represents *knowledge* and $\mathcal{C}_\theta$ is a boulomaic operator that represents *will*.

[20] This version of **Eq** is inspired by an equivalent rule formulated in G.E. Hughes, M.J. Cresswell, *A New Introduction to Modal Logic*, op. cit., p. 32.

most important. However, we could deal with states of affairs involving more than one situation, say $\alpha(p, q)$, but this is dispensable for our purposes.

For ease of reading, we establish a standard interpretation for each *wff*, according to the following semantic in natural language, which can be considered as a set of abbreviations:

θ ≔ "God";
$P(p)$ ≔ "p is the case";
$B(\theta)$ ≔ "θ is divine";
$\delta(p)$ ≔ "p is good";
$\xi(p)$ ≔ "p is evil";
$\mathcal{C}_\theta\, \alpha(p)$ ≔ "God wills the state of affairs $\alpha(p)$";
$\mathcal{W}_\theta\, \alpha(p)$ ≔ "God knows the state of affairs $\alpha(p)$."

In the following, we give an account of some of the divine attributes in order to provide a formal treatment of the questions raised in section 1.

2.2. The Attributes of God

We begin by presenting the definition of divinity:

Def. 1. (Divinity). $B(\theta) :\leftrightarrow (WW \wedge NM \wedge WM \wedge DB)$

(God is divine *iff* He is omniscient, infallible, omnipotent, and omnibenevolent.)

The following definitions aim at formally expressing some relations that characterize the divine attributes of classical theism according to our approach. They are essentially proposed by Nieznański and they have the advantage of their precision and of a certain correspondence with the traditional concepts in the historical debate.[21]

Def. 2. $WW :\leftrightarrow \forall p\, (\alpha(p) \rightarrow \mathcal{W}_\theta\, \alpha(p))$

(God is omniscient *iff* for all situations, if a state of affairs is the case, then God knows it.)

[21] Cf. E. Nieznański, *Aksjomatyczne ujęcie problemu teodycei*, op. cit., pp. 204–205; E. Nieznański, *Elements of Modal Theodicy*, op. cit., pp. 255–256.

Def. 3. $NM :\leftrightarrow \forall p\ (\mathcal{W}_\theta\ \alpha(p) \rightarrow \alpha(p))$

(God is infallible *iff*, for all situations, if God knows a state of affairs, then it is the case.)

Def. 4. $WM :\leftrightarrow \forall p\ (\mathcal{C}_\theta\ \alpha(p) \rightarrow \alpha(p))$

(God is omnipotent *iff*, for all situations, if God wills a state of affairs, then it is the case.)

Def. 5. $DB :\leftrightarrow \forall p\ (\mathcal{C}_\theta\ P(p) \rightarrow \delta(p))$

(God is omnibenevolent *iff*, for all situations, if God wills a situation to be the case, then such situation is good.)

Thus, the most relevant divine attributes are defined in order to approach the problem of evil, namely, omniscience, infallibility, omnipotence, and omnibenevolence.

As we have stated in the last section, **N3** is composed of only three axioms. These axioms are not difficult to assume in the context of the problem of evil; we believe that to some extent they correspond to what Mackie intends to do when he proposes the contemporary version of the problem of evil.[22]

A1. $B(\theta)$

(God is divine.)

A2. $\neg\forall p\ (P(p) \rightarrow \delta(p))$

(Not all situations are such that, if a situation is the case, then such situation is good.)[23]

A3. $\forall p\ (\delta(p) \rightarrow \neg\xi(p))$

(For all situations, if a situation is good, then it is not evil.)

[22] Mackie states some of the attributes of God, affirms that there is evil, and relates evil situations to good ones. Then, he argues for the incompatibility of those concepts. See J.L. Mackie, *Evil and Omnipotence*, op. cit., pp. 200–201.

[23] An equivalent formulation of A2 would be $\exists p\ (P(p) \wedge \neg\delta(p))$, i.e., there is at least one situation that is the case but is not good.

As an immediate consequence of Def. 1 and A1, the validity of the four attributes considered is established:

T1. $WW \wedge NM \wedge WM \wedge DB$

(God is omniscient, infallible, omnipotent, and omnibenevolent.)

Proof.

1. $B(\theta)$	[A1]
2. $B(\theta) :\leftrightarrow WW \wedge NM \wedge WM \wedge DB$	[Def. 1]
3. $WW \wedge NM \wedge WM \wedge DB$	[**PC**, 1, 2][24]

□

The following theorems are corollaries of T1 and can be easily obtained:

T1.1. $\forall p\,(\alpha(p) \rightarrow \mathcal{W}_\theta\,\alpha(p))$ □

(For all situations, if a state of affairs is the case, then God knows such a state of affairs.)

T1.2. $\forall p\,(\mathcal{W}_\theta\,\alpha(p) \rightarrow \alpha(p))$ □

(For all situations, if God wills a state of affairs, then such state of affairs is the case.)

T1.3. $\forall p\,(\mathcal{C}_\theta\,\alpha(p) \rightarrow \alpha(p))$ □

(For all situations, if God knows a state of affairs, then such state of affairs is the case.)

T1.4. $\forall p\,(\mathcal{C}_\theta\,P(p) \rightarrow \delta(p))$ □

(For all situations, if God wills some situation to be the case, then such situation is good.)

[24] We use **PC** to indicate the use of some theorems or results from the Propositional Calculus.

Summing up, in all situations, and for arbitrary states of affairs, if a state of affairs is the case, then God knows it, for He is omniscient; if God knows a state of affairs, then it is the case, for God is infallible in His knowledge; if God wills a state of affairs, then it is the case, for he is omnipotent; and, finally, as God is omnibenevolent, if he wills a state of affairs, then it is good.

In this regard, David Hume asks: why is there evil in the world? As he remarks in his classical statement on the problem of evil:

> Epicurus's old questions are yet unanswered. Is he willing to prevent evil, but not able? then is he impotent. Is he able, but not willing? then is he malevolent. Is he both able and willing? whence then is evil?[25]

Hume seems to assume that if God does not will a state of affairs, then such a state of affairs is not the case. But this is equivalent to saying that if a state of affairs is the case, then God wills such a state of affairs. Such a statement is the reciprocal of the attribute of omnipotence and is not necessarily valid. In fact, we will prove that such a hypothesis – which is here called *religious determinism*, or, simply, *determinism* (**DET1**) – is not a theorem in **N3**.

Another version of determinism is more related to God's omniscience, since it supposes that if a state of affairs is the case, then He wills such a state of affairs (**DET2**). Surprisingly, it can be easily proved in **N3** that the negation of **DET2** is derived from the negation of **DET1**.

In this way, we formally state two deterministic hypotheses, which are refuted in **N3**.

(DET1). $\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$

> (For all situations, if a situation is the case, then God wills such situation to be the case.)

(DET2). $\forall p\ (\mathcal{W}_\theta\, P(p) \rightarrow \mathcal{C}_\theta\, P(p))$

> (For all situations, if God knows a situation to be the case, then God wills such situation to be the case.)

Next, we proceed to refute these two versions of religious determinism.

[25] D. Hume, *Dialogues Concerning Natural Religion*, in: *David Hume: Dialogues Concerning Natural Religion in Focus*, ed. S. Tweyman, London 2013 [1779], p. 273.

2.3. Religious Determinism Defeated

As we have already said, **N3** is a minimal system, in the sense of assuming a very small number of axioms in comparison to **N1** and **N2**, in order for us to be able to propose a solution to the famous logical problem of evil. Therefore, we will present some results about the existence of contingent situations and the fact that there are states of affairs that are not willed by God, but that are permitted. It can be considered that such results show the possibility of an investigation on free will. However, this is outside the scope of this article.

That said, we now proceed to formally refute the two versions of determinism.

T2 (¬DET1). $\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$

(Not all situations are such that, if a situation is the case, then God wills such situation to be the case.)

Proof.

1. $\neg\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[Hip][26]
2. $\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[**PC**, 1]
3. $P(p) \rightarrow \mathcal{C}_\theta\, P(p)$	[2, Spec][27]
4. $\mathcal{C}_\theta\, P(p) \rightarrow \delta(p)$	[T1.4, Spec]
5. $P(p) \rightarrow \delta(p)$	[**PC**, 3, 4]
6. $\forall p\ (P(p) \rightarrow \delta(p))$	[Gen, 5][28]
7. $\neg\forall p\ (P(p) \rightarrow \delta(p))$	[A2]
8. $\neg\neg\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[¬Hip, 6, 7]
9. $\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[**PC**, 8] □

T3 (¬DET2). $\neg\forall p\ (\mathcal{W}_\theta\, P(p) \rightarrow \mathcal{C}_\theta\, P(p))$

(Not all situations are such that if God knows a situation to be the case, then God wills such a situation to be the case.)

[26] Hypothesis.
[27] Specification Rule.
[28] Generalization Rule.

Proof.

1. $\forall p\ (\mathcal{W}_\theta\, P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[Hip]
2. $\mathcal{W}_\theta\, P(p) \rightarrow \mathcal{C}_\theta\, P(p)$	[1, Spec]
3. $P(p) \rightarrow \mathcal{W}_\theta\, P(p)$	[T1.1, $\alpha(p)/P(p)$, Spec]
4. $\mathcal{W}_\theta\, P(p) \rightarrow P(p)$	[T1.2, $\alpha(p)/P(p)$, Spec]
5. $P(p) \leftrightarrow \mathcal{W}_\theta\, P(p)$	[**PC**, 3, 4]
6. $P(p) \rightarrow \mathcal{C}_\theta\, P(p)$	[Eq, 5 in 2]
7. $\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[Gen, 6]
8. $\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[T2]
9. $\neg\forall p\ (\mathcal{W}_\theta\, P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[**PC**, 9]

□

Thus, we easily refuted the two versions of determinism which are fundamental for the discussion about the logical problem of evil. To solve the question of determinism without needing many previous results is a remarkable characteristic of **N3**.

2.4. Further Consequences: God, Values and Determinism

Given the results of the previous section, we now explore some elementary results with respect to the problem of evil in itself. We think these theorems and definitions are self-explanatory: they, in conjunction, provide a framework to reconsider whether there is compatibility between the existence of evil and the existence of God.

T4. $\forall p\ (\xi(p) \rightarrow \neg\delta(p))$

(For all situations, if a situation is evil, then such situation is not good.)

Proof. Easily deduced from A3, by contraposition. □

T5. $\forall p\ (\neg\delta(p) \rightarrow \neg\mathcal{C}_\theta\, P(p))$

(For all situations, if a situation is not good, then it is not the case that God wills it to be the case.)

Proof. Easily deduced from T1.4, by contraposition. □

T6. $\forall p\ (\xi(p) \rightarrow \neg\mathcal{C}_\theta\, P(p))$

(For all situations, if a situation is evil, then it is not the case that God wills it to be the case.)

Proof.

1. $\xi(p) \rightarrow \neg\delta(p)$	[T4, Spec]
2. $\neg\delta(p) \rightarrow \neg\mathcal{C}_\theta\, P(p)$	[T5, Spec]
3. $\xi(p) \rightarrow \neg\mathcal{C}_\theta\, P(p)$	[**PC**, 1, 2]
4. $\forall p\ (\xi(p) \rightarrow \neg\mathcal{C}_\theta\, P(p))$	[Gen, 3]

□

T7. $\forall p\ \neg\mathcal{C}_\theta\, (\alpha(p) \wedge \neg\alpha(p))$

(For all situations, it is not the case that God wills some contradiction.)

Proof.

1. $\mathcal{C}_\theta\, (\alpha(p) \wedge \neg\alpha(p)) \rightarrow (\alpha(p) \wedge \neg\alpha(p)))$	[T1.3, $\alpha(p)/(\alpha(p) \wedge \neg\alpha(p))$, Spec]
2. $\neg(\alpha(p) \wedge \neg\alpha(p)) \rightarrow \neg\mathcal{C}_\theta\, (\alpha(p) \wedge \neg\alpha(p))$	[**PC**, 1]
3. $\neg(\alpha(p) \wedge \neg\alpha(p))$	[**PC**-Theorem]
4. $\neg\mathcal{C}_\theta\, (\alpha(p) \wedge \neg\alpha(p))$	[MP, 3, 2]
5. $\forall p\ \neg\mathcal{C}_\theta\, (\alpha(p) \wedge \neg\alpha(p))$	[Gen, 4]

□

T8. $\forall p\ \mathcal{C}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$

(All situations are such that God wills non-contradictions.)

Proof.

1. $\neg(\alpha(p) \wedge \neg\alpha(p))$ [**PC**-Theorem]
2. $\mathcal{C}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$ [Nec, 1]
3. $\forall p\ \mathcal{C}_\theta \neg(\alpha(p) \wedge \neg\alpha(p))$ [Gen, 2]

□

Permission is defined in **N3** as the dual operator of $\mathcal{C}_\theta$:

Def. 6 (Permission). $\mathcal{D}_\theta\ \alpha(p) :\leftrightarrow \neg\mathcal{C}_\theta \neg\alpha(p)$

(God permits a state of affairs *iff* He does not will the opposite.)

T9. $\forall p\ (\mathcal{C}_\theta\ \alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \neg\alpha(p))$

(For all situations, God wills a state of affairs *iff* He does not permit the opposite.)

Proof.

1. $\mathcal{D}_\theta \neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta \neg\neg\alpha(p)$ [Def. 6, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{D}_\theta \neg\alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\ \alpha(p)$ [**PC**, 1]
3. $\mathcal{C}_\theta\ \alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \neg\alpha(p)$ [**PC**, 2]
4. $\forall p\ (\mathcal{C}_\theta\ \alpha(p) \leftrightarrow \neg\mathcal{D}_\theta \neg\alpha(p))$ [Gen, 3]

□

T10. $\forall p\ (\alpha(p) \rightarrow \mathcal{D}_\theta\ \alpha(p))$

(For all situations, if a state of affairs is the case, then it is permitted by God.)

Proof.

1. $\mathcal{C}_\theta \neg\alpha(p) \rightarrow \neg\alpha(p)$	[T1.3, $\alpha(p)/\neg\alpha(p)$, Spec]
2. $\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$	[Def. 6, Spec]
3. $\alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p)$	[**PC**, 1, 2]
4. $\forall p\ (\alpha(p) \rightarrow \mathcal{D}_\theta \alpha(p))$	[Gen, 3]

□

T11. $\neg\forall p\ (\mathcal{D}_\theta \alpha(p) \rightarrow \alpha(p))$

(Not all situations are such that, if God permits some state of affairs, then it is the case.)

Proof.

1. $\forall p\ (\mathcal{D}_\theta \alpha(p) \rightarrow \alpha(p))$	[Hip]
2. $\mathcal{D}_\theta \alpha(p) \rightarrow \alpha(p)$	[Spec, 1]
3. $\mathcal{D}_\theta \alpha(p) \leftrightarrow \neg\mathcal{C}_\theta\neg\alpha(p)$	[Def. 6, Spec]
4. $\neg\mathcal{C}_\theta\neg\alpha(p) \rightarrow \alpha(p)$	[Eq, 3 in 2]
5. $\neg\alpha(p) \rightarrow \mathcal{C}_\theta\neg\alpha(p)$	[**PC**, 4]
6. $\alpha(p) \rightarrow \mathcal{C}_\theta \alpha(p)$	[5, $\alpha(p)/\neg\alpha(p)$, **PC**]
7. $P(p) \rightarrow \mathcal{C}_\theta P(p)$	[6, $\alpha(p)/P(p)$]
8. $\forall p\ (P(p) \rightarrow \mathcal{C}_\theta P(p))$	[Gen, 7]
9. $\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta P(p))$	[T2]
10. $\neg\forall p\ (\mathcal{D}_\theta \alpha(p) \rightarrow \alpha(p))$	[¬Hip, 1]

□

A *contingent* state of affairs, according to the philosophical tradition, is one such that this state of affairs and its complementary are both possible, or, as we define here, permitted by God:

Def. 7. $K(p) :\leftrightarrow (\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$

(A situation is contingent *iff* God permits it to be or not to be the case.)

T12. $\forall p\, (K(p) \leftrightarrow (\neg \mathcal{C}_\theta P(p) \wedge \neg \mathcal{C}_\theta \neg P(p)))$

(For all situations, a situation is contingent *iff* neither God wills such situation to be the case, nor wills it not to be the case.)

Proof.

1. $K(p) \leftrightarrow (\mathcal{D}_\theta P(p) \wedge \mathcal{D}_\theta \neg P(p))$	[Def. 7]
2. $\mathcal{D}_\theta P(p) \leftrightarrow \neg \mathcal{C}_\theta \neg P(p)$	[Def. 6, Spec]
3. $\mathcal{C}_\theta P(p) \leftrightarrow \neg \mathcal{D}_\theta \neg P(p)$	[T9, Spec]
4. $\mathcal{D}_\theta \neg P(p) \leftrightarrow \neg \mathcal{C}_\theta P(p)$	[**PC**, 3]
5. $K(p) \leftrightarrow (\neg \mathcal{C}_\theta P(p) \wedge \neg \mathcal{C}_\theta \neg P(p))$	[Eq, 2 $\wedge$ 4 in 1, **PC**]
6. $\forall p\, (K(p) \leftrightarrow (\neg \mathcal{C}_\theta P(p) \wedge \neg \mathcal{C}_\theta \neg P(p)))$	[Gen, 5]

□

The following corollaries are easily deduced from T12:

T12.1. $\forall p\, (K(p) \leftrightarrow \neg(\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p)))$ □

(For all situations, a situation is contingent *iff* either it is not the case that God wills such situation to be the case or He wills such situation not to be the case.)

T12.2. $\exists p\, (K(p) \leftrightarrow \neg \forall p\, (\mathcal{C}_\theta P(p) \vee \mathcal{C}_\theta \neg P(p)))$ □

(There is a contingent situation *iff* it is not the case that, for all situations, either God wills a situation to be the case or He is opposed to that.)

T12.3. $\forall p\ (K(p) \leftrightarrow (\neg \mathcal{C}_\theta\, P(p) \wedge \mathcal{D}_\theta\, P(p)))$ □

(For all situations, a situation is contingent *iff* either it is not the case that God wills such situation to be the case or He permits such situation not to be the case.)

Now, we proceed to prove that at least one contingent situation exists. In order to do this, we use the following theorem:

T13. $\neg\forall p\ (\mathcal{C}_\theta\, \alpha(p) \vee \mathcal{C}_\theta \neg\alpha(p))$

(Not all situations are such that either God wills a state of affairs or its opposite.)

Proof.

1.	$\forall p\ (\mathcal{C}_\theta\, \alpha(p) \vee \mathcal{C}_\theta \neg\alpha(p))$	[Hip]
2.	$\mathcal{C}_\theta\, \alpha(p) \vee \mathcal{C}_\theta \neg\alpha(p)$	[1, Spec]
3.	$\neg\mathcal{C}_\theta\, \alpha(p) \rightarrow \mathcal{C}_\theta \neg\alpha(p)$	[**PC**, 2]
4.	$\mathcal{C}_\theta \neg\alpha(p) \rightarrow \neg\alpha(p)$	[T1.3, $\alpha(p)/\neg\alpha(p)$, Spec]
5.	$\neg\mathcal{C}_\theta\, \alpha(p) \rightarrow \neg\alpha(p)$	[**PC**, 3, 4]
6.	$\alpha(p) \rightarrow \mathcal{C}_\theta\, \alpha(p)$	[**PC**, 5]
7.	$\forall p\ (\alpha(p) \rightarrow \mathcal{C}_\theta\, \alpha(p))$	[Gen, 6]
8.	$\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[7, $\alpha(p)/P(p)$]
9.	$\neg\forall p\ (P(p) \rightarrow \mathcal{C}_\theta\, P(p))$	[T2]
10.	$\neg\forall p\ (\mathcal{C}_\theta\, \alpha(p) \vee \mathcal{C}_\theta \neg\alpha(p))$	[¬Hip, 1]

□

T14. $\exists p\ K(p)$

(There is at least one situation that is contingent.)

Proof.

1. $\exists p\ (K(p) \leftrightarrow \neg\forall p\ (\mathcal{C}_\theta\, P(p) \vee \mathcal{C}_\theta \neg P(p)))$	[T12.2]
2. $\neg\forall p\ (\mathcal{C}_\theta\, P(p) \vee \mathcal{C}_\theta \neg P(p))$	[T13, $\alpha(p)/P(p)$]
3. $\exists p\ K(p)$	[**PC**, 1, 2]

□

T15. $\forall p\ ((\xi(p) \wedge \mathcal{D}_\theta\, P(p)) \rightarrow K(p))$

(For all situations, if a situation is evil, and God permits it, then such situation is contingent.)

Proof.

1. $\neg\forall p\ ((\xi(p) \wedge \mathcal{D}_\theta\, P(p)) \rightarrow K(p))$	[Hip]
2. $\exists p\ \neg((\xi(p) \wedge \mathcal{D}_\theta\, P(p)) \rightarrow K(p))$	[**FOL**, 1][29]
3. $\neg((\xi(p) \wedge \mathcal{D}_\theta\, P(p)) \rightarrow K(p))$	[**FOL**, 2]
4. $(\xi(p) \wedge \mathcal{D}_\theta\, P(p)) \wedge \neg K(p)$	[**PC**, 3]
5. $\neg K(p)$	[**PC**, 4]
6. $\xi(p) \wedge \mathcal{D}_\theta\, P(p)$	[**PC**, 4]
7. $\xi(p)$	[**PC**, 6]
8. $\mathcal{D}_\theta\, P(p)$	[**PC**, 6]
9. $\xi(p) \rightarrow \neg\mathcal{C}_\theta\, P(p)$	[T6, Spec]
10. $\neg\mathcal{C}_\theta\, P(p)$	[MP, 7, 9]
11. $\neg\mathcal{C}_\theta\, P(p) \wedge \mathcal{D}_\theta\, P(p)$	[**PC**, 10, 8]
12. $K(p) \leftrightarrow (\neg\mathcal{C}_\theta\, P(p) \wedge \mathcal{D}_\theta\, P(p))$	[T12.3, Spec]
13. $K(p)$	[**PC**, 12, 11]
14. $\neg\neg\forall p\ ((\xi(p) \wedge \mathcal{D}_\theta\, P(p)) \rightarrow K(p))$	[¬Hip, 5, 13]

[29] We use **FOL** to indicate the use of some theorems or results from the First-Order Logic.

15. $\forall p\ ((\xi(p) \wedge \mathcal{D}_\theta P(p)) \rightarrow K(p))$ [Gen, 15]

□

Theorem T15 is equivalent to $\forall p\ ((\xi(p) \wedge \neg\mathcal{C}_\theta\neg P(p)) \rightarrow K(p))$. It means, therefore, that if there is evil in the world and God seems to not will the opposite, then this very situation is contingent and thus does not depend on his good will. In conjunction with the refutation of **DET1** and **DET2**, these theorems explain how the existence of evil can be consistent with the attributes of God. The answer is quite simple: as determinism fails (T4 and T5), if there is evil in the world, God does not will such an evil to be the case (T6), but if He permits evil, then it must be contingent (T15).

Finally, we present a semantics for **N3**.

2.5. Semantics of N3

As usual, the model for the system **N3** is a structure $\langle W, R, D, V\rangle$, where W is a set of possible worlds; $R \subseteq W^2$ is a relation of accessibility; D is the domain of objects; and V: $WFF \times W \rightarrow \{0,1\}$ is a function of valuation, where $\phi, \psi \in WFF$ and $w, w' \in W$, determined by an assignment μ, such that, for each variable p of $\mathfrak{L}_{N3}$, $\mu(p) \in D$. In particular, the valuations for the modal operators $\mathcal{W}_\theta$ and $\mathcal{C}_\theta$ are given by the following conditions:

(i) $V(\mathcal{W}_\theta \phi, w) = 1$ *iff* $V(\phi, w) = 1$

(ii) If $V(\mathcal{C}_\theta \phi, w) = 1$ then $V(\phi, w') = 1$ for every $w' \in W$ such that wRw'.

Let $W = \{w_0, w_1, \ldots, w_n, \ldots\}$, where $n \in \mathrm{N}$, be a set of possible worlds, $R = W^2$, and $D = \mathbb{N} \cup \{-1\}$, and for every assignment μ we fix that $\mu(\theta) = 0$. The function of valuation V is such that:

$V(B) = \{(0, w_n): n \in \mathbb{N}\}$;

$V(\xi) = \{(-1, w_n): n \in \mathbb{N}\}$;

$V(P) = V(B) \cup V(\xi) \cup \{(2n, w_n): n \in \mathbb{N}\}$;

$V(\delta) = V(P) - V(\xi)$.

We proceed to show that $\mathfrak{M} = \langle W, R, D, V\rangle$, thus defining a model for the axioms of **N3**. It is worth noting that this model does not explicitly distinguish objects from situations. Furthermore, we can say that an object p satisfies a predicate P_i in a possible world w *iff* $(\mu(p), w) \in V(P_i)$.

Let us begin with axioms **A2** and **A3**, because we use **A2** to show **A1**.

[A2:] $\mathfrak{M} \vDash \forall p\ (\delta(p) \rightarrow \neg\xi(p))$

Proof. Since $V(\delta) = V(P) - V(\xi)$, then, $V(\delta) \cap V(\xi) = \emptyset$. Thus, for every p, it is not the case that $(\mu(p), w) \in V(\delta)$ and $(\mu(p), w) \in V(\xi)$. Thus, for every p, it is not the case that $V(\delta(p), w) = 1$ and $V(\xi(p), w) = 1$. Then, by De Morgan's law, we have that, for every p, $V(\delta(p), w) = 0$ or $V(\xi(p), w) = 0$. Therefore, for every p, $V(\delta(p), w) = 0$ or $V(\neg\xi(p), w) = 1$. So, for every p, $V(\delta(p) \rightarrow \neg\xi(p), w) = 1$. It follows that $V(\forall p\ (\delta(p) \rightarrow \neg\xi(p)), w) = 1$. □

[A3:] $\mathfrak{M} \vDash \neg\forall p\ (P(p) \rightarrow \delta(p))$

Proof. We have that $V(\neg\forall p\ (P(p) \rightarrow \delta(p)), w) = 1$ *iff* $V(\forall p\ (P(p) \rightarrow \delta(p)), w) = 0$. The latter is the case, *iff* there is an object p, such that $V(P(p), w) = 1$ and $V(\delta(p), w) = 0$. We have that $(\text{-}1, w) \in V(\xi) \cap V(P)$, because $(\text{-}1, w_n) \in V(\xi)$, for every $n \in \mathrm{N}$, and $V(\xi) \subset V(P)$. Thus, since V and μ are well-defined functions, there is an object p in the domain, such that $\mu(p) = \text{-}1$. Therefore, we have an object p, such that $(\mu(p), w) \in V(P)$ and $(\mu(p), w) \in V(\xi)$. Since $V(\delta) \cap V(\xi) = \emptyset$, so we have that $(\mu(p), w) \in V(P)$ and $(\mu(p), w) \notin V(\delta)$. But this implies that $V(P(p), w) = 1$ and $V(\delta(p), w) = 0$. Therefore, $V(\neg\forall p(P(p) \rightarrow \delta(p), w) = 1$. □

Now, let us consider **A1**. Since $B(\theta)$ is obtained through a conjunction, that is, $WW \wedge NM \wedge WM \wedge DB$, it is enough to show that its constituent formulas are true according to the interpretation considered:

[A1:] $\mathfrak{M} \vDash WW \wedge NM$

Proof. $WW \wedge NM$ is equivalent to $\forall p\ (\alpha(p) \leftrightarrow \mathcal{W}_\theta\, \alpha(p))$. By the condition (i) above, we have that, for every p, $V(\mathcal{W}_\theta\, \alpha(p), w) = 1$ *iff* $V(\alpha(p), w) = 1$. Therefore, we have that $V(\forall p\ (\alpha(p) \leftrightarrow \mathcal{W}_\theta\, \alpha(p)), w) = 1$. Therefore, $WW \wedge NM$ is true in $\mathfrak{M}$. □

$\mathfrak{M} \vDash WM$

Proof. By definition, *WM* is equivalent to $\forall p\ (\mathcal{C}_\theta\ \alpha(p) \rightarrow \alpha(p))$. By the condition (ii) above, we have that if $V(\mathcal{C}_\theta\ \alpha(p), w) = 1$ then $V(\alpha(p), w) = 1$, since R is reflexive, and, consequently, wRw. Thus, $V(\forall p\ (\mathcal{C}_\theta\ \alpha(p) \rightarrow \alpha(p)), w) = 1$. Therefore, *WM* is true in $\mathfrak{M}$. □

$\mathfrak{M} \vDash DB$

Proof. The only restriction to assignments μ in $\mathfrak{M}$ is that $\mu(\theta) = 0$, since θ is the only distinguished object in **N3**. Therefore, we can consider two assignments, μ_i and μ_j, and a pair of objects, p and q, with $p \neq q \neq \theta$, such that

$(\mu_i(p), w_m) \notin V(P)$ and $(\mu_i(q), w_m) \in V(\xi)$; and

$(\mu_j(q), w_m) \notin V(P)$ and $(\mu_j(p), w_m) \notin V(\xi)$;

for some $m, n \in \mathbb{N}$, and $m \neq n$.

But that means that

(*) $V(P(p), w_m) = 0$ and $V(\xi(q), w_m) = 1$; and

(**) $V(P(q), w_n) = 0$ and $V(\xi(p), w_n) = 1$;

for some $m, n \in \mathbb{N}$, and $m \neq n$.

Now, let us suppose that an r exists such that $V(\mathcal{C}_\theta\ P(r), w) = 1$ but $V(\delta(r), w) = 0$. Then, from **A3**, we have that $V(\mathcal{C}_\theta P(r), w) = 1$ and $V(\xi(r), w) = 1$. So, from condition (ii), we have that (***) $V(P(r), w') = 1$ for every $w' \in W$, and $V(\xi(r), w) = 1$. But, if we took $p = r$ and $w_n = w$ in (*) and (**), we would have that $V(P(r), w_n) = 0$ and $V(\xi(r), w) = 1$, for some $m \in \mathrm{N}$, contradicting (***). Thus, $V(\forall p\ (\mathcal{C}_\theta\ P(p) \rightarrow \delta(p)), w) = 1$. Therefore, *DB* is true in $\mathfrak{M}$. □

Thereby, we concluded that $\mathfrak{M} \vDash B(\theta)$, and, therefore, $\mathfrak{M}$ is a model for **N3**.

The meta-theory is assumed to be consistent. Thus, when we give a model (in this case, a set-theoretical model) for theory **N3**, we conclude that it is also consistent. Otherwise, if it were possible to derive a contradiction in **N3**, it would imply that there would be a contradiction in the model. That is, the consistency

of **N3** is conditioned by the consistency of the meta-theory and/or, in this case, also by the consistency of the set theory. Thus, from the fact that ¬**DET1** and ¬**DET2** are theorems of **N3** it is reasonable to conclude that **DET1** and **DET2** are not theorems of **N3**.

Final Remarks

In this article, we described a first-order modal system called **N3**, a system which aims at dealing with religious determinism and the logical problem of evil. On the one hand, if the results are correct, then we have an answer to these questions, now stated in formal terms with clarity and precision. On the other hand, **N3** establishes its results with many fewer assumptions than **N1**, **N2**, or those original systems developed by Nieznański,[30] and in a widely recognized formal language. Both outcomes, together, provide a narrower response to the logical problem of evil through the refutation of determinism, possibly offering a new pathway to the solution of this debate.

For all that has been shown, the results provide a characterization of many issues involving the logical problem of evil. Furthermore, in conjunction, they provide a framework for reconsidering whether there is compatibility between the existence of evil and the existence of God. We hope that it deals satisfactorily with the allegation of religious determinism; we think that such results provide a detailed approach that must not be ignored, as they provide a relevant response to such difficult questions. At least, for those involved in the mainstream discussion, even if the solutions available do not fulfil their pretension, there is one more solution at hand, which tackles the question in an innovative way, and with tools that may be more precise than those of just natural language.

Finally, as we remarked in the Introduction, we believe that the usage of formal systems may provide several advantages in solving philosophical and theological problems. "Standing on the shoulders" of Bocheński, of the Cracow Circle, and of Nieznański as well, we think that our approach can be considered

[30] We recognize that, despite affirming that the goal of his systems was to deal with the problem of evil, Nieznański had much more in mind than just dealing with this question. In his systems, he deals with God's will and a number of dispositions (permission, opposition, causality, responsibility), exhibiting a full-blown treatment of these "divine properties."

an example of logic of religion, and, as such, we hope that it contributes to the advancement of the field.

Acknowledgements

We thank the Editors of this volume for the initiative. Also, we thank the anonymous reviewers for helpful suggestions on earlier versions of this paper.

This research was made possible through the support of grant no. 61108 ("Formal Approaches to Philosophy of Religion and Analytic Theology") from the John Templeton Foundation. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the John Templeton Foundation.

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Summary

Edward Nieznański developed two logical systems to deal with the problem of evil and to refute religious determinism. However, when formalized in first-order modal logic, two axioms of each system contradict one another, revealing that there is an underlying minimal set of axioms enough to settle the questions. In this article, we develop this minimal system, called **N3**, which is based on Nieznański's contribution. The purpose of **N3** is to solve the logical problem of evil through the defeat of a version of religious determinism. On the one hand, these questions are also addressed by Nieznański's systems, but, on the other hand, they are obtained in **N3** with fewer assumptions. Our approach can be considered a case of logic of religion, that is, of logic applied to religious discourse, as proposed by Józef Maria Bocheński; in this particular case, it is a discourse in theodicy, which is situated in the context of the philosophy of religion.

Key words: logical problem of evil, theodicy, first-order modal logic, logic of religion, Edward Nieznański

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Logical Analysis of the Concept of Beauty

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1. Introduction

Besides his remarkable contributions to logic and its history, and as part of his intense philosophical activity, Józef Maria Bocheński carried out noteworthy investigations on concepts belonging to the most varied domains of knowledge. Among these, we highlight his input into discussions on the concepts of *authority* and *free society*. In these, Bocheński undertakes logical analyses in a methodology he developed himself, and that allows him to work with great conceptual precision and to obtain results relevant for understanding these crucial relationships in human societies.

Inspired by his methodology, we will carry out a logical analysis of the concept of beauty. We are not, however, aiming for the same level of depth achieved by Bocheński in his works. This is a first attempt at a logical treatment of an extremely slippery and controversial concept. Therefore, we do not wish to get involved in the philosophical dispute about what beauty means. Our approach is less pretentious than that, but we consider the results presented here to be relevant for those participating in discussions on philosophical aesthetics, consistently, from the perspective of classical logic. In fact, we analyze the truth values of some basic propositions about beauty based on certain assumptions on the

domains of objects and subjects and two formal definitions concerning famous beliefs related to the concept of *beauty*. For this purpose, we use a kind of semantic neutrality that allows us to analyze other concepts whose extensions and definitions satisfy the same assumptions. We consider the most interesting result to be the one affirming that, even in a relativistic perspective, it is necessary to affirm the existence of an object that is either beautiful or non-beautiful and that is universally recognized as such by all subjects in the domain.

In section 1.1, we briefly discuss Bocheński's approach in his analyses of the concepts of authority and free society, which inspired our method. In section 2, we give a brief overview of how beauty was conceptualized in philosophy throughout history and of the classic discussions about it. In section 3, we undertake a logical analysis of the concept of beauty. In 3.1, we introduce two formal and very general definitions of beauty and give some examples of how they can formally capture the definitions and insights of some philosophical traditions. In 3.2, we discuss the conditions imposed on the domain of subjects. In 3.3, we present the assumptions on the extension of the concept of beauty as well as the recognition of beautiful and non-beautiful objects by the subjects. In 3.4, we give twelve basic propositions about beauty, based on the possible generalizations of beauty as a binary relation. In the same section, we assign truth values to these basic propositions. We discuss the possible valuations of open truth-value propositions according to the two definitions given in 3.1. Finally, we present some conclusions in section 4.

1.1. On Bocheński's Analysis of the Concepts of Authority and Free Society

Bocheński's philosophical activity could be succinctly described as philosophical analysis.[1] In particular, his analytical papers can be considered a part of the Polish analytic philosophy programme, inaugurated by Kazimierz Twardowski and Jan Łukasiewicz. The latter's proposal of conceptual analysis strongly affected Bocheński's approach.[2]

[1] An interesting discussion of Bocheński's method of philosophical analysis can be found in M. Lechniak, *Bocheński's Method of Philosophical Analysis and Contemporary Applied Ontology*, "Studies in East European Thought" 2013, Vol. 65, pp. 17–26.

[2] It is noteworthy that Bocheński is one of the founders of the Cracow Circle, which unfortunately was unable to implement its entire programme due to the dissolution of the group during World War II. As Jan Woleński remarks: "Let me note that Bocheński's works about the concepts of

Bocheński's innovation in his analysis of the concept of authority can be summed up in two main points. First, he distinguishes between *epistemic authority* and *deontic authority*. Second, such a distinction makes it possible to explain the general form of authority through a triadic relation between a carrier, the subject, and a field. Thus, "$A(x, y, \alpha)$" means that "x (carrier) is an authority for y (subject) in the field α." Based on this formula, Bocheński introduces a series of interesting generalizations.[3]

Despite the interesting results obtained in his analysis of the concept of authority, it is his paper entitled *The Concept of the Free Society* that interests us most.[4] In the paper, Bocheński presents a logical analysis of the concept of a free society, which presupposes the triadic relation of deontic authority. Thus, his investigation depends only on an intuitive interpretation of authority, which allows him to present, through generalizations of the formula "$F(x, j)$" (for "x is free in the domain j"),[5] twelve types of *a priori* societies with respect to freedom.

In this way, Bocheński proceeds to give successive definitions of a free society that culminates in what is, in our opinion, a successful definition, through the rigour of logical symbolism. His approach shows that it is possible to employ tools that are simple from a logical point of view, but that provide very precise and original results in philosophy.

We will proceed in a similar way to Bocheński, investigating *beauty* as a binary relation.[6] However, we will evaluate the truth values of some basic propositions about beauty, which is possible thanks to certain assumptions about the domains of objects and subjects.

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authority and the logic of religion are perhaps the extreme realization of the ideology of the Cracow Circle" – J. Woleński, *Polish Attempts to Modernize Thomism by Logic (Bocheński and Salamucha)*, "Studies in East European Thought" 2003, Vol. 55, No. 4, p. 312, n. 21.

[3] He discusses the concept of authority and its various types in the following works: J. Bocheński, *On Authority*, "Comunicaciones Libres" 1964, Vol. 5, pp. 45–46; J. Bocheński, *The Logic of Religion*, New York, NY 1965; J. Bocheński, *An Analysis of Authority*, in: *Authority*, ed. F. Adelmann, The Hague 1974, pp. 56–85; J.M. Bocheński, *Was ist Autorität? Einführung in die Logik der Autorität*, Freiburg 1974; J. Bocheński, *On Authority*, "South African Journal of Philosophy" 1989, Vol. 8, No. 2, pp. 61–65.

[4] J. Bocheński, *The Concept of the Free Society*, "The Monist" 1986, Vol. 69, No. 2, pp. 207–215.

[5] The formula $F(x, j)$ is defined by Bocheński as $\sim (\exists y)\, A(y, x, j)$, which means "$x$ is not subject to any deontic authority in j" (cf. J. Bocheński, *The Concept of the Free Society*, op. cit., p. 207).

[6] Ibid. Beauty can also be considered as a triadic relation, so that the object (x) is beautiful to the subject (α) in context (k). However, we will not discuss this further in this paper.

To the best of our knowledge, Bocheński himself does not seem to have undertaken investigations in philosophical aesthetics, nor even carried out any analyses of the concept of beauty. Therefore, we believe that applying an approach inspired by Bocheński's methodology can be a relevant and original contribution to the logic of beauty and to formal aesthetics.

2. On the Concept of Beauty

Throughout most of the history of aesthetics, beauty has been its most emblematic topic. From Plato to Arthur Danto, beauty has been one of the main concepts in aesthetics and one of the most controversial themes in the history of Western philosophy.[7] In modern aesthetics, though, the concept of *beauty* carries specific connotations and has taken on greater importance with the consolidation of the modern system of the arts since the early 1700s.[8]

The concept of beauty has engendered many philosophical problems, such as those related to the definition of beauty, the nature of beauty, to whether beauty is related to content instead of only formal elements, and to the superiority of one type of beauty over another.[9] The topic of the nature of beauty has been debated by many well-known aesthetes. Philosophers understood beauty, among many other ways, as a type of *pleasure*,[10] as a characteristic of objects based on a given

[7] *Hippias major* 281a–304a; A. Danto, *O abuso da beleza*, São Paulo 2015.

[8] See P. Kristeller, *The Modern System of the Arts: A Study in the History of Aesthetics. Part I*, "Journal of the History of Ideas" 1951, Vol. 12, No. 4, p. 499; P. Kristeller, *The Modern System of the Arts: A Study in the History of Aesthetics. Part II*, "Journal of the History of Ideas" 1952, Vol. 13, No. 1, p. 17.

[9] We can see this, for example, in the different positions that characterized the traditional German idealist debate between Immanuel Kant and Georg Wilhelm Friedrich Hegel about the superiority of one type of beauty over another (for Kant, natural beauty over artistic beauty and, for Hegel, artistic beauty over natural beauty). See I. Kant, *Critique of Judgement*, trans. J.H. Bernard, New York, NY 1951; G.W.F. Hegel, *Cursos de estética I*, São Paulo 2001. For further reading, see G. Rebec, *Natural vs. Artistic Beauty*, "The Journal of Philosophy, Psychology and Scientific Methods" 1905, Vol. 2, No. 10, pp. 253–260.

[10] G. Santayana, *The Sense of Beauty*, New York, NY 1955, pp. 11–13; Thomas Aquinas, ST I, q. 5, a. 4. "[B]eauty relates to the cognitive faculty; for beautiful things are those which please when seen" (ST I, q. 5, a. 4, ad 1). The concept of beauty as a type of pleasure in Aquinas (something that "pleases when seen") is reinforced and deeply discussed by Christopher Sevier in *Aquinas on Beauty*, Lanham 2015.

ratio or on symmetry,[11] or even as something related to morality, truth and happiness.[12] But, along with the problems of the definition and nature of *beauty*, another key philosophical issue concerning *beauty* is its subjectivity/objectivity.

Since antiquity, philosophers have been discussing the subjectivity/objectivity of beauty. On the one hand, for some philosophers, such as Plato, beauty consists in an immanent property contained in the *form* of things, as he exposes in the *Hippias major*. In other words, for Plato beauty is a property, a characteristic, of objects: therefore, beauty should necessarily be objective. On the other hand, for philosophers such as David Hume, beauty is related to *taste*, as he states in his essay *Of the Standard of Taste* (1757). For Hume, beauty consists much more in a value linked to taste and subjectivity than in a property which is recognized by all subjects as beautiful, as thought by Plato.[13]

Thus, since *beauty* has been discussed by many thinkers throughout the history of ideas, can we determine whether it is an attribute, or whether it is relational? Is beauty an objective property of objects or a judgement/valuation about a personal representation of objects? If *beauty* is an attribute/property, then we can say that an object is beautiful if, and only if, beauty can be represented as a unary predicate. If *beauty* is relative to an individual (that is, it is a value or

[11] F. Hutcheson, *An Inquiry into the Original of Our Ideas of Beauty and Virtue*, Indianapolis, IN 2004, p. 29; Vitruvius, *On Architecture*, trans. F. Granger, Cambridge 1970, pp. 26–27. For Hutcheson, an object is called *beautiful* when it "seems to be in a compound Ratio of Uniformity and Variety; so that where the Uniformity of Bodies is equal, the Beauty is as the Variety; and where the Variety is equal, the Beauty is as the Uniformity" – F. Hutcheson, *An Inquiry into the Original of Our Ideas of Beauty and Virtue*, op. cit., p. 29. For Vitruvius, the concept of *beauty* is directly linked to the concept of *venustas* – which, on a basic level, refers to concepts such as those of symmetry and harmony.

[12] J. Armstrong, *The Secret Power of Beauty*, London 2004. For John Armstrong, *beauty* is found in acquiring "spiritual value (truth, happiness, moral ideals) at home in a material setting (rhythm, line, shape, structure) and in a way that, while we contemplate the object, the two seem inseparable" (ibid., p. 163).

[13] Some philosophers, such as Immanuel Kant and Roger Scruton, also opposed the absolute subjectivity of beauty as a value, judgement or feeling, affirming, in this way, that beauty consists in a judgement which is rationally founded and has a degree of objectivity (objectivity that makes it capable of being at least communicable). See I. Kant, *Crítica da faculdade de julgar*, trans. F. Costa Mattos, Petrópolis 2016, pp. 132–136; I. Kant, *Observations on the Feeling of the Beautiful and Sublime*, in: *Observations on the Feeling of the Beautiful and Sublime and Other Writings*, eds. P. Frierson, P. Guyer, New York, NY 2011, pp. 13–18; R. Scruton, *Beauty*, New York, NY 2009, pp. 195–197.

a judgement), then we can state that an object is beautiful if, and only if, beauty can be represented as a binary relation.

Perhaps there is a little less disagreement about the meaning of a free society in the civilized world (at least in terms of the universal and theoretical acceptance of human rights) than in the field of philosophical aesthetics, which has been a challenge since its beginning. However, in this paper, instead of trying to present a precise definition of *beauty*, we will focus on the analysis of the truth values of some basic propositions about beauty, so that they can be interpreted according to different positions on the matter. And, surprisingly, such an analysis leads to interesting conclusions concerning propositions about the existence of universally recognized objects, either beautiful or non-beautiful.

It is possible that there are other aesthetic values besides beauty and ugliness, and so we will not assume that non-beautiful is a synonym for ugly. According to some authors, such as Danto, there are objects that, in certain contexts, for example, have no aesthetic value, being neutral or just useful.[14]

3. Logical Analysis of the Concept of Beauty

In this section, we introduce two formal definitions of beauty, from the perspective of recurring patterns found in various philosophical definitions in which these concepts come to be expressed with reasonable clarity. Below, we present the fundamental background which, according to us, is needed to perform a logical analysis of the concept of beauty. For this purpose, we establish the conditions that must be satisfied by the domain of subjects, and we identify assumptions that will allow us to evaluate the truth values of basic propositions about beauty.

3.1. Two Formal Definitions of Beauty

Regardless of how beauty is defined or how the meaning of the expression "x is beautiful" is established, we can identify the following patterns.

[14] The Dadaist artist Marcel Duchamp, for example, once said that his readymades had the fundamental objective of having "no beauty, no ugliness, nothing particularly aesthetic" about them; they were supposed to be "as nonsensical as possible." The topic of Duchamp's nonsensical/non-aesthetic objects has been addressed by authors such as Arthur Danto or Calvin Tomkins. See A. Danto, *O abuso da beleza*, op. cit., pp. 108–109; C. Tomkins, *The World of Marcel Duchamp*, New Jersey, NJ 1972, pp. 38–39.

In a *universalistic* sense, one could consider that

(i) *to say that x is beautiful means to say that x is beautiful for* ***everyone****.*

In turn, in *relativistic* perspective, we would claim that

(ii) *to say that x is beautiful means to say that x is beautiful for* ***someone****.*

In such cases, we can distinguish the use of "beautiful" as a unary predicate (i) and as a binary relation (ii). Substituting "$B(x)$" for "x is beautiful" and "$\beta(x, \alpha)$" for "x is beautiful for the subject α," we can rewrite (i) and (ii) as follows:

(i) $B(x) \equiv \forall\alpha\, \beta(x, \alpha)$;

(ii) $B(x) \equiv \exists\alpha\, \beta(x, \alpha)$.

We will call formulas (i) and (ii) the *universalistic* and *relativistic* definitions of beauty, respectively. We must stress that in this case the meaning of the terms universalistic and relativistic is determined by the use of universal and existential quantifiers in the formulas. Interpretations of such formulas may contemplate several underlying philosophies of beauty. For the logical analysis carried out here, we do not need to adopt any specific philosophical position regarding beauty. Thus, if a certain philosophical conception is such that it admits the efficacy of classical logic and assigns the truth value true to our assumptions, then the conclusions presented here must be consequences in its philosophical system.

The universalistic and relativistic definitions establish two connections between the predicate "beautiful" and the binary relation "beautiful for a subject." What the latter means formally or informally depends on the theory of beauty underlying the adopted semantics. For example, Marcia Eaton claims that cognitive theories of beauty take the following form:

(CT) *x is beautiful if, and only if, attention to intrinsic properties of x yields pleasurable perceptual experiences in an informed observer who is observing x.*[15]

There are two possibilities of interpretation here. If the domain of subjects consists only of informed observers, then (CT) can be interpreted as a universalistic definition and "beautiful" in "x is beautiful" is a predicate. If the domain

[15] M. Eaton, *Beauty and Ugliness In and Out of Context*, in: *Contemporary Debates in Aesthetics and Philosophy of Art*, ed. M. Kieran, Oxford 2006, p. 46.

of subjects includes non-informed observers, then (CT) can be interpreted from a relativistic perspective, so that "*x* is beautiful" means tacitly "*x* is beautiful to an observer/subject."

In both cases, we could claim that the second member (the *definiens*) of the equivalence in (CT) establishes the binary relation in question. Thus, one possibility of symbolizing (CT) is

$$\text{(CT)} \quad B(x) \equiv \forall\alpha\,(OI(\alpha, x) \,\&\, PP(x, \alpha)),$$

where "$OI(\alpha, x)$" stands for "α is an informed observer who is observing x"; "$PP(x, \alpha)$" stands for "attention to intrinsic properties of x yields pleasurable perceptual experiences in α"; and "$\beta(x, \alpha)$" is equivalent to "$OI(\alpha, x) \,\&\, PP(x, \alpha)$." This last equivalence would allow us to consider (CT) a definition of the universalistic type (i).

In any case, the values of formulas (i) and (ii) depend on the domains of objects and subjects.

With that in mind, some examples of universalists could be:

(a) *objectivists*, who claim that beauty is objective and that a beautiful object is recognized as such by all subjects.

(b) *conventionalists*, who think that beauty is a convention, recognized by everyone involved in it. In this case, the domain of subjects must be restricted.

(c) *cognitivists*, who believe that a beautiful object can be recognized by every subject with the necessary knowledge. In this case, the domain of subjects must also be restricted.

On the other hand, some examples of relativists could be:

(a′) *subjectivists*, who argue that the foundation of an object's aesthetic value lies in the subject. In this case, it only makes sense to say that something is beautiful for the subject himself.[16]

[16] The most-known defenders of this position are Hume and Kant; see D. Hume, *Of the Standard of Taste*, in: *Essays Moral, Political, and Literary*, ed. E.F. Miller, Indianapolis, IN 1987; I. Kant, *Critique of Judgement*, op. cit.; I. Kant, *Observations on the Feeling of the Beautiful and Sublime*, op. cit.; I. Kant, *Crítica da faculdade de julgar*, op. cit. Hume, for example, once affirmed that "beauty is no quality in things themselves: It exists merely in the mind which contemplates them; and each mind perceives a different beauty" – D. Hume, *Of the Standard of Taste*, op. cit., p. 139. It is important to note that, even being subjectivists, neither Kant nor Hume have defended that beauty is not at least a communicable value – which expresses, to some extent, a minimal degree of objectivity in their conception of the subjectivity of beauty.

(b′) *aesthetic relativists*, who assume that aesthetic value depends solely on an individual or on the social group. In this case, the domain of subjects is partitioned, and something is beautiful only as a reference to a subset of such a domain.

(c′) *strong cognitivists*, who argue that no landscape untouched by humans can be non-beautiful. In this case, it is sufficient that there is at least one subject informed well enough to perceive the beauty of natural objects.[17]

Since there are some inaccuracies in the attempts to define beauty in different schools of thought, the examples above depend on a certain interpretation and manipulation of the domains of objects and subjects. Thus, for example, a *cognitivist* who is more like a *relativist* can also be considered a *universalist* in a restricted domain. In other words, categories (b), (c), (b′), and (c′) have a certain degree of neutrality with respect to the formal definitions presented above, even though they tend to belong to one side (as a universalist or a subjectivist). The only categories listed here which are necessarily characterized as universalists or relativists in a strong sense are, respectively, (a) and (a′). That is, it is impossible for an individual to be simultaneously an *objectivist* and a *subjectivist*. Otherwise, an individual can simultaneously be a *cognitivist* and a *relativist* or a *strong cognitivist* and a *universalist*. This is because, in these two last cases, the properties of being a *cognitivist* or a *stronger cognitivist* are not necessarily linked to the property of being a universalist or a relativist, respectively.

In the following, we discuss the domain of subjects who can enter into relations with objects, which can be judged as beautiful or non-beautiful.

3.2. The Domain of Subjects

As a criterion for establishing the domain of subjects, we will consider it to be a set of *skilled agents*. In what follows, we will clarify what it means to say skilled agents.

It is expected that for a subject to be able to make aesthetic judgements about objects, he or she must be able to access it in some way. Let us assume that each

[17] This specific category of cognitivists (which, here, we named *strong cognitivists*) is presented by Eaton in *Beauty and Ugliness In and Out of Context*, op. cit., pp. 47–48. An example of an author that defended this position is Allen Carlson – see his *Nature and Positive Aesthetics*, "Environmental Ethics" 1984, Vol. 6, No. 5, p. 5.

object has certain properties that require specific skills/abilities to be accessed. It follows, then, that we will assume that the subjects have such skills/abilities.

As is well known, discussions on beauty end up orbiting around two classic possibilities: *beauty is in the object*, or *beauty is in the eye of the beholder*.[18] If beauty is *objective*, then we can say that the object is beautiful due to its intrinsic properties. If beauty is *subjective*, then the object is *subjectively beautiful*, that is, it is beautiful to someone. On one hand, if we admit that beauty is objective, the beautiful object can only be perceived as beautiful by someone who can access its intrinsic properties. On the other hand, if we admit that it is subjective, then for an object to be considered *subjectively beautiful* by a subject, it is necessary that the object can be accessed in some way, which corresponds to access to some properties of that object.

Let us take the example of *visual objects*. In order to be considered *visually beautiful*, it is necessary to be visually perceived. It can be said that people incapable of visual perception do not have the minimum skill/ability necessary to consider something *visually beautiful*, whether beauty is objective or subjective.

In addition, to be a skilled agent a subject depends on certain psychological, cognitive, and epistemic states. For our goal, we stipulate that the subjects have a minimally stable cognition, that is, that they are individual persons without very serious mental or psychological disorders. Thus, people who have conditions so extreme as to prevent them from considering anything beautiful are not in the domain of subjects.

Once the conditions for the domain of subjects are stated, we can establish some assumptions that will guide our analysis.

3.3. The Assumptions

From a very general point of view on the concept of beauty, we will adopt the following propositions as assumptions.

A1. *Not everything falls within the scope of the concept of beauty*;

A2. *Something falls within the scope of the concept of beauty*;

[18] These expressions are related, respectively, to the possibilities of objectivity and subjectivity in beauty. The phrase "beauty is in the eye of the beholder," referring to the subjectivity of beauty, was first used by Margaret Hungerford, in her novel *Molly Bawn* (1878).

A3. *Every subject recognizes at least one object as beautiful and at least one object as non-beautiful.*

Assumption A1 guarantees that the extension of the concept is not the totality of things. Assumption A2, in turn, expresses the fact that the extension is not empty. Both avoid the trivialization (totality or emptiness) of the concept of beauty. The third assumption guarantees the same for beauty as a binary relation. Such assumptions avoid two extreme cases, namely, saying that everything is beautiful or saying that nothing is beautiful.

Extreme cases can occur in certain artificial situations in which the concept of beauty would not be treated in its true scope. For example, suppose that the domain of objects is a collection formed by the Parthenon, the Niagara Falls, and Duchamp's Fountain. Furthermore, let the domain of subjects be determined by, say, four individuals who consider the three "objects" to be beautiful. In this case, we would have that the extension of the local concept of beauty would be the whole domain, both in the relativistic and in the universalistic sense. Now, if the domain is a set of people who do not consider any of the three objects beautiful, then the extension of the local concept would be empty. Notwithstanding these possibilities, it does not seem to be the state of affairs in reality, when we consider the domains formed by all things and all subjects (possibly within a certain period of time).[19]

Rewriting the assumptions with symbols, we obtain the following formulas:

A1. $\neg\forall x\, B(x)$ (which is equivalent to either $\neg\forall x\forall\alpha\, \beta(x, \alpha)$ or $\neg\forall x\exists\alpha\, \beta(x, \alpha)$);

A2. $\exists x\, B(x)$ (which is equivalent to either $\exists x\forall\alpha\, \beta(x, \alpha)$ or $\exists x\exists\alpha\, \beta(x, \alpha)$);

A3. $\forall\alpha\exists x\exists y\, (\beta(x, \alpha)\ \&\ \neg\, \beta(y, \alpha))$.

Based on these assumptions, we can evaluate the truth values of basic propositions about beauty.

[19] In order to avoid problems such as those generated by Russell's Paradox, we could consider the domain of objects to be the set of all real or actual objects, assuming that a set is not a real being. In this way, it seems reasonable to speak of the totality of (actual) things.

3.4. Twelve Basic Propositions on Beauty as a Binary Relation

Using the formula $\beta(x, \alpha)$ it is possible to establish twelve different basic formulas on beauty. Since we have made assumptions about the domains of objects and subjects, we can determine the truth value of most of these propositions. In fact, only in the case of two pairs of contradictory formulas are the truth values *a priori* open. Thus, given the context of a reflection in aesthetics, we can identify the advantages and disadvantages of assuming certain statements to be true. Next, we present the twelve possibilities of generalization of quantifiers on the binary relation.

We proceed now to list the different basic propositions on beauty. There are, first, two generalizations of one of the variables, either "x" or "α". Thus, we need to determine the number of formulas obtained from the combination *quantifier-variable-quantifier-variable-*$\beta(x, \alpha)$. This is a very simple problem in combinatorics; the solution is 2 x 2 x 2 x 1 = 8 formulas. Similarly, we will have eight quantified formulas on $\neg\ \beta(x, \alpha)$, which results in a total of sixteen generalizations. However, the order of variables for two equal quantifiers is irrelevant. Thus, there are twelve different possible generalizations. Table 1 presents these twelve possibilities, where we omit $\beta(x, \alpha)$ for brevity.

Table 1

1. $\forall x \forall \alpha$ and $\forall \alpha \forall x$	7. $\exists x \exists \alpha \neg$ and $\exists \alpha \exists x \neg$
2. $\exists \alpha \forall x$	8. $\forall \alpha \exists x \neg$
3. $\forall x \exists \alpha$	9. $\exists x \forall \alpha \neg$
4. $\exists x \forall \alpha$	10. $\forall x \exists \alpha \neg$
5. $\forall \alpha \exists x$	11. $\exists \alpha \forall x \neg$
6. $\exists x \exists \alpha$ and $\exists \alpha \exists x$	12. $\forall x \forall \alpha \neg$ and $\forall \alpha \forall x \neg$

The "dodecagon" (Figure 1) below presents all twelve formulas, and it indicates some implications through arrows (for example, formula 1 implies 2, 3, 4, etc; formula 2 implies 3; and so on). Contradictory formulas can be identified by their diametrically opposite position according to the analogue clock. Thus, formulas 1 and 7 are contradictory; 2 and 8 are contradictory; 3 and 9 are contradictory; and so on.

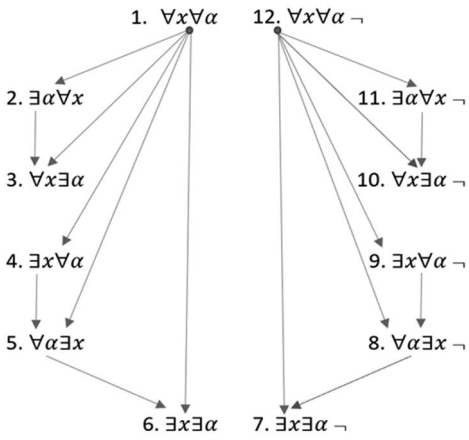

Figure 1

In the following, we present natural-language interpretations of the twelve basic formulas and we also evaluate each one.

1. $\forall x \forall \alpha\ \beta(x, \alpha)$

   All objects are beautiful for all subjects.

This means that everything is beautiful for everyone. If this formula is true, then the extension is the universe, but this contradicts assumption A1. Moreover, it contradicts assumption A3 as well. Therefore, formula 1 is **false**.

2. $\exists \alpha \forall x\ \beta(x, \alpha)$

   There is at least one subject for whom all objects are beautiful.

Formula 2 says that someone recognizes beauty in everything. But it contradicts assumption A3. Therefore, formula 2 is **false**.

3. $\forall x \exists \alpha\ \beta(x, \alpha)$

   For every object, there is at least one subject who recognizes its beauty.

According to the **universalistic** definition of beauty, formula 3 can be **true** or **false**. In the **relativistic** sense, it is **false**, because it contradicts assumption A1.

4. $\exists x \forall \alpha\ \beta(x, \alpha)$

There is at least one object that is beautiful for all subjects.

Formula 4 means that there exists at least one case of *universal beauty*, that is, something that is beautiful for everyone, an object that is universally recognized as beautiful. In the **universalistic** sense, it is **true**. It can be **true** or **false** according to the **relativistic** definition of beauty.

5. $\forall \alpha \exists x\ \beta(x, \alpha)$

For all subjects, there is at least one object that is beautiful.

This formula affirms that everyone recognizes that something is beautiful. According to assumption A3, formula 5 is **true**. Otherwise, we would have to assume that someone from the domain of subjects thinks that everything is non-beautiful.

6. $\exists x \exists \alpha\ \beta(x, \alpha)$

There is at least one object that is beautiful for at least one subject.

This means that something is beautiful for someone. We could say that this fact is empirically verifiable, since it is easy to find someone who considers at least one object to be beautiful. In any case, formula 6 follows from formula 5. Therefore, it is **true**.

7. $\exists x \exists \alpha\ \neg\ \beta(x, \alpha)$

There is at least one object that is not beautiful for at least one subject.

Such a formula says that something is non-beautiful for someone. It follows from assumption A3. Furthermore, it is contradictory to formula 1, which is false. Thus, formula 7 is **true**.

8. $\forall \alpha \exists x\ \neg\ \beta(x, \alpha)$

For all subjects, there is at least one object that is not beautiful.

It also follows from assumption A3. In addition, it is contradictory to formula 2, which is false. Therefore, formula 8 is **true**.

9. $\exists x \forall \alpha\ \neg\ \beta(x, \alpha)$

There is at least one object that is not beautiful for all subjects.

Analogously to formula 4, this formula means that there exists at least one case of *universal non-beauty*, that is, something that is non-beautiful for everyone. In the **universalistic** sense, it can be **true** or **false**. It is **true** according to the **relativistic** definition, because if its contradictory were true (formula 3), then the concept of beauty would be trivialized (everything would be beautiful).

10. $\forall x \exists \alpha \neg \beta(x, \alpha)$

    For every object, there is at least one subject who does not recognize its beauty.

This formula is **false** in the **universalistic** sense, since it is the contradictory of formula 4 (otherwise the extension of the concept would be empty). Just like formula 4, it can be **true** or **false** according to the **relativistic** definition of beauty.

11. $\exists \alpha \forall x \neg \beta(x, \alpha)$

    There is at least one subject for whom all objects are not beautiful.

It contradicts assumption A3. Furthermore, it is the contradictory of formula 5, which is true. It follows that its truth value is **false**.

12. $\forall x \forall \alpha \neg \beta(x, \alpha)$

    All objects are not beautiful for all subjects.

This formula means that nothing is beautiful. It contradicts assumption A2 and formula 6, which is true. Therefore, formula 12 is **false**.

The following table summarizes the valuations established so far.

Table 2

1. $\forall x \forall \alpha$	F	7. $\exists x \exists \alpha \neg$	T
2. $\exists \alpha \forall x$	F	8. $\forall \alpha \exists x \neg$	T
3. $\forall x \exists \alpha$	?	9. $\exists x \forall \alpha \neg$	?
4. $\exists x \forall \alpha$	?	10. $\forall x \exists \alpha \neg$	?
5. $\forall \alpha \exists x$	T	11. $\exists \alpha \forall x \neg$	F
6. $\exists x \exists \alpha$	T	12. $\forall x \forall \alpha \neg$	F

We now know which basic propositions about beauty are true, regardless of which of the two formal definitions of beauty is adopted. Next, we will evaluate the two pairs of propositions with open truth-values, considering each definition.

3.5. Evaluating the Open Truth-Value Propositions According to the Two Formal Definitions of Beauty

In this section, we present four possible formal theories about beauty, based on the valuation of formulas with open truth-values.

3.5.1. The Universalistic Context

First, let us suppose that the universalistic definition of beauty applies. Then,

(i) $B(x) \equiv \forall\alpha\, \beta(x, \alpha)$

is the formal definition of beauty to be considered.

From the valuations made in section 3.4, we have that formula 4, that is, $\exists x\forall\alpha\, \beta(x, \alpha)$, is true. This means that there is at least one object that is beautiful for all subjects, which we call a *universal beauty*. This was already expected, because, in the universalistic context, every object to which the predicate beautiful is applied is a *universal beauty*, since to be beautiful means to be beautiful for everyone. Thus, Table 3 indicates the truth values of the basic formulas in this context.

Table 3

1. $\forall x\forall\alpha$	F	7. $\exists x\exists\alpha\neg$	T
2. $\exists\alpha\forall x$	F	8. $\forall\alpha\exists x\neg$	T
3. $\forall x\exists\alpha$	?	9. $\exists x\forall\alpha\neg$	?
4. $\exists x\forall\alpha$	T	10. $\forall x\exists\alpha\neg$	F
5. $\forall\alpha\exists x$	T	11. $\exists\alpha\forall x\neg$	F
6. $\exists x\exists\alpha$	T	12. $\forall x\forall\alpha\neg$	F

One of the two contradictory formulas (3 and 9) must be true. Next, let us analyze each case.

1. $\forall x\exists\alpha\, \beta(x, \alpha)$ is true.

If formula 3 is true, then we a have a *universalistic-relativistic theory of beauty*. It is universalistic in the sense that it assumes that every beautiful object is universally recognized as such. It is also relativistic because each object is beautiful for at least one of the subjects, that is, every object is relatively beautiful.

2. $\exists x \forall \alpha \neg \beta(x, \alpha)$ is true.

If formula 9 is true, then we a have a *universalistic theory of beauty and non-beauty*. In this case, in addition to a universal beauty, we also have a universal non-beauty, that is, an object universally recognized as non-beautiful.

3.5.2. The Relativistic Context

Now, let us consider the relativistic definition of beauty. In this case,

(ii) $B(x) \equiv \exists \alpha \, \beta(x, \alpha)$

is the corresponding formal definition of beauty.

Again, from the valuations made in 3.4, we have that formula 9, that is, $\exists x \forall \alpha \neg \beta(x, \alpha)$, is true. At this point we arrive at an unexpected consequence, namely, the affirmation of the existence of something universally recognized by the subjects as non-beautiful. Apparently, in a relativistic context, universal statements of this kind should be avoided, that is, it does not seem very relativistic to assume propositions stating that a certain judgement is made by all subjects in the domain. This fact is surprising, especially if we consider the domain of subjects to be all living people today or even all human beings. The alternative would be to conclude that formula 3 is true, but, in this case, the concept would be trivialized, and so everything would be beautiful. Table 4 indicates the truth values of the basic formulas in this relativistic context.

Table 4

1. $\forall x \forall \alpha$	F	7. $\exists x \exists \alpha \neg$	T
2. $\exists \alpha \forall x$	F	8. $\forall \alpha \exists x \neg$	T
3. $\forall x \exists \alpha$	F	9. $\exists x \forall \alpha \neg$	T
4. $\exists x \forall \alpha$	?	10. $\forall x \exists \alpha \neg$	?
5. $\forall \alpha \exists x$	T	11. $\exists \alpha \forall x \neg$	F
6. $\exists x \exists \alpha$	T	12. $\forall x \forall \alpha \neg$	F

1. $\exists x \forall \alpha\ \beta(x, \alpha)$ is true.

If formula 4 is true, then we have here a *relativistic-universalistic theory of beauty*. It is relativistic since it assumes that for an object to be beautiful it is enough that at least one subject considers it so. It is universalistic because in this context there is a universal beauty.

2. $\forall x \exists \alpha \neg \beta(x, \alpha)$ is true.

If that is the case, then what we have here is a set of propositions that configure something that we could call the *theory of universal non-beauty*. It is so because, on the one hand, there would be an object universally recognized as non-beautiful, and, on the other hand, each object would be considered non-beautiful for at least one subject.

4. Conclusions

In short, using an approach inspired by Bocheński, we were able to carry out a logical analysis of the concept of beauty, in order to investigate the truth values of certain basic propositions regarding beauty. For this purpose, we considered two formal definitions of beauty that establish two connections between beauty as a predicate and beauty as a binary relation. In each case, that is, in the universalistic and relativistic contexts, there remains a set of contradictory propositions with undetermined truth values. Thus, we conclude that one can assume *a priori* four types of theories of beauty, namely, the universalistic-relativistic theory of beauty, the universalistic theory of beauty and non-beauty, the relativistic-universalistic theory of beauty, and the theory of universal non-beauty. In the universalistic context, it is not surprising that we must conclude that a universal beauty exists, since this follows from the very definition of beauty and the non-emptiness of the domain of objects. In the case of the relativistic context, however, the conclusion that we should assume the existence of a universal non-beauty does not seem obvious. Furthermore, we believe that for the purpose of a relativistic investigation in philosophical aesthetics, it is more appropriate to adopt the *relativistic-universalistic theory of beauty*. This is because in the relativistic context there remains the choice between the assumption of a *universal beauty* and the conclusion that *everything is relatively non-beautiful*. If the latter is the case,

then the true basic propositions are essentially immediate consequences of the assumptions made here, plus two universal propositions about the non-beautiful. Furthermore, if non-beauty is a synonym of ugliness, then that would mean *there is something that is considered ugly for everyone* and *everything is relatively ugly*. And that would be more like a *theory of ugliness* than a *theory of beauty*.

Therefore, the *relativistic-universalistic theory of beauty* is the most interesting in the relativistic context. However, it is nonetheless surprising that the assumption about the existence of a universal beauty is preferable. One is not expected, in a relativistic context, to assume universal propositions of this nature, except for the proposition asserting that everything is relative.

Furthermore, we note that it is possible to build a formal first-order theory corresponding to each of the four theories discussed. In this case, formulas 5, 6, 7, and 8 would be theorems common to the four theories, assuming only one of the formal definitions of beauty and the formal versions of the assumptions as axioms. Finally, it is worth mentioning that the approach proposed here can be used to analyze other concepts.

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Summary

Since antiquity, defining the concept of *beauty* has been a struggle for philosophers. Many raised questions related to the objectivity/subjectivity of beauty, which then became fundamental to the understanding of issues in philosophical aesthetics. In this context, our paper provides a logical analysis of the concept of beauty, which includes both universalistic and relativistic perspectives. Based on a methodology inspired by Józef Maria Bocheński's logical analyses of the concepts of authority and free society, we intend to present some unexpected results derived from popular beliefs and to propose solutions concerning this issue.

Key words: beauty, logic, aesthetics, formal approaches

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ISSN 0860-3839
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Gerland's *Dialectica* and Paraconsistency

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1. Introduction

The present state of paraconsistent logic attests to significant development, and its maturity permits a critical historical analysis of this advance, having in view the appreciation of its historical roots and stages of formation. The aim of our general research project consists in studying how a truly paraconsistent perspective was constituted throughout the history of logic, as well as how logical principles, rules, and systems have expressed the various contemporary concepts of paraconsistency. This article is a part of this project. It is also inspired by the approach of Józef Maria Bocheński, and in a way it follows his scientific programme. In particular, we have adopted here his thesis according to which the formal aspects of logical theory are essential, decisive, and indispensable to a good historiography of logic. The notion of logical consequence and the use of principles and rules in the obtaining of valid inferences are central in this context.[1]

Analyzing the historical precedents of paraconsistent logic before the 20th century, we can identify some unanswered questions, among which are the following: was there knowledge of logical rules and principles that allowed, in some

[1] J.M. Bocheński, *A History of Formal Logic*, trans. I. Thomas, New York, NY 1970, pp. 2–23.

contexts, for inconsistency to be treated without trivialization? If such principles were known, how were these "proto-principles" stated, and in what way can they be related to the logical-paraconsistent results and rules known today? In light of these questions, we may ask ourselves if logical principles and rules according to which not everything may be deduced from a contradiction, or something may be rejected, were conceived and evoked within certain contexts and theoretical traditions.

In our research, we have studied key authors of the mediaeval period, focusing on primary sources, results and scholarly literature that are related to contradiction and the principle *ex falso sequitur quodlibet* and are pertinent to the history of paraconsistent logic. Scholastic logic is marked by a close proximity of logic, grammar, and metaphysics. There was an intense debate at this time about the validity of the principle *ex falso sequitur quodlibet* (or *ex impossibili sequitur quodlibet*, or *ex contradictione sequitur quodlibet*)[2] in the context of theories of implication, the doctrine of topics, and obligational disputations. The analysis of positions for and against the *ex falso* is essential to the history of the paraconsistent approach in scholastic logic and in Western thought. It is important to observe here that Bocheński, in his celebrated *Formale Logik* (1956), affirms that the *ex falso* is a mediaeval contribution to logic – "This [the Aristotelian discussion of valid syllogisms based on false premises in *An. pr.* B2, 53b7–10] is not yet the scholastic principle *ex falso sequitur quodlibet*, but only the assertion that one can

[2] Motivated by a historical analysis, specifically by the discussions of the *ex falso* by several thinkers during the Middle Ages, in this paper – though we recognize the distinct logical nuances – we consider the expression *ex falso sequitur quodlibet* to embrace such principles as special types of the *ex falso*. As far as we know, the first thinker to use the expression *idem esse ex contradictione* was John of Salisbury, alluding to the position of Adam of Balsham's school in the debate; see Ioannis Saresberiensis, *Metalogicon (Metalogicus)*, in: *Patrologia Latina*, Vol. 199, ed. J.-P. Migne, Paris 1815–1875, 928C–D. Also, as far as we know, it was Chris Mortensen, in his well-known book *Inconsistent Mathematics*, Dordrecht 1995, p. 2, who used the expression *ex contradictione quodlibet* in the context of paraconsistency. In 1996, Andrés Bobenrieth used the expression *ex contradictione sequitur quodlibet*; see A. Bobenrieth Miserda, *Inconsistencias ¿Por qué no? Un estudio filosófico sobre la lógica paraconsistente*, Bogotá 1996, p. 103. Maria Luisa Dalla Chiara mentions *ex absurdo sequitur quodlibet*; see M.L. Dalla Chiara, *Logica*, Milano 1974, p. 27. In fact, the expression *ex contradictione sequitur quodlibet* had been previously used in the literature by several other logicians, such as, for instance, Else M. Barth and Erik C. Krabbe, in 1982; see E.M. Barth, E.C. Krabbe, *From Axiom to Dialogue*, Berlin 1982, p. 167.

form syllogisms in which one or both premises are false, the conclusion true"[3] – an observation which has been corroborated by later scholarship.

As far as we know, the first thinkers to discuss the validity of what came to be referred to as the principle *ex falso sequitur quodlibet* were Gerland of Besançon and Peter Abelard. John of Salisbury (1159) attributes to Adam of Balsham (the head of the *parvipontani*) the thesis that "from a contradiction follows the same" (*idem esse ex contradictione*), which is generalized by the *ex falso*.[4] From the 13th century on, there was an intense debate concerning the validity of some *consequentiae*, and one finds generations of logicians quarrelling over the legitimacy of the *ex falso*. In fact, we consider that the first author to explicitly argue against the *ex falso* under a *lato sensu* paraconsistent approach was Peter Abelard.[5] In a forthcoming work we will present our analysis of his position.

In this paper, we analyze the role of the *Dialectica* of Gerland of Besançon in the rising of discussion about the *ex falso* in the 12th century, and we interpret his position as contrary to the acceptance of the principle. We consider Gerland one of the earliest authors to prepare the path and to properly discuss the role of the *ex falso sequitur quodlibet*, making it central in the philosophical context of the time.

As this author and his work are not well known among philosophers, we decided to provide important details in this regard. In section 2, we discuss the identity of the author of the *Dialectica*. In section 3, we outline the content of the work, with emphasis on the theory of topics and on three aspects of logical theory which are important for our discussion. In section 4, we analyze Gerland's concept of consequence, fundamental to logic and to our analysis concerning paraconsistency, presenting his semantic clauses for the veracity and the falsity of a consequence. Next, we present very basic notions about paraconsistent logic and paraconsistent theories, also introducing the key concepts of relevant logics. Finally, we conclude by interpreting Gerland's position as contrary to the accept-

[3] J.M. Bocheński, *A History of Formal Logic*, op. cit., p. 98.

[4] See L. Minio-Paluello, *Twelfth Century Logic: Texts and Studies*, Vol. 1: *Adam Balsamiensis Parvipontani Ars Disserendi (Dialectica Alexandri)*, Roma 1956; and Ioannis Saresberiensis, *Metalogicon* (*Metalogicus*), op. cit., 928C–D.

[5] See E.L. Gomes, I.M.L. D'Ottaviano, *Para além das colunas de Hércules, uma história da paraconsistência: de Heráclito a Newton da Costa* [Beyond the Columns of Hercules, a History of Paraconsistency: From Heraclitus to Newton da Costa], Campinas 2017, pp. 164–181.

ance of the *ex falso sequitur quodlibet*, including him as a defender of what is nowadays considered a paraconsistent approach in the broad sense.

In accord with Yukio Iwakuma, we assume that Gerland of Besançon, a contemporary of Abelard, wrote his *Dialectica* some years before Abelard's writings were produced. Although we are aware of the polemic concerning the authorship of some of Abelard's works, we have decided not to discuss this question here and have simply assumed that Abelard's *Editio super Porphyrium* was known during the period we are analyzing and is posterior to the publication of Gerland's *Dialectica*.[6]

In the development of our work we have used Garlandus Compotista's *Dialectica*, edited by Lambertus Marie De Rijk.[7] In this paper, we have opted to use quotations from Eleonore Stump and Ivan Boh, well-known scholars of mediaeval philosophy and logic, who dedicated themselves directly to the study of the development of the logical-philosophical theories of the period and in particular to the study of Gerland's contributions. We quote their accurate translations in specific excerpts on the notion of consequence that we consider significant for our analysis of a possible paraconsistent approach in the *Dialectica* of Gerland of Besançon. However, as far as we know, Stump and Boh do not present approaches concerning either relevance or paraconsistency in their analyses of the *Dialectica*, and we do not know of any other authors that have explicitly analyzed Gerland's approach as being paraconsistentist *lato sensu*. It is precisely such an analysis that we consider the specific contribution of this article.

2. Gerland of Besançon

Gerland "the Computist" was for a time, due to the attribution of De Rijk,[8] considered the author of the *Dialectica*, an important treatise of scholastic logic and

[6] Y. Iwakuma, *"Vocales," or Early Nominalists*, "Traditio" 1992, Vol. 47, pp. 53–54. See, e.g., C.J. Martin, *A Note on the Attribution of the "Literal Glosses" in Paris, BnF, lat. 13368 to Peter Abaelard*; and M. Cameron, *Abelard's Early Glosses: Some Questions*, both in: *Arts du langage et théologie aux confins des XIe–XIIe siècles: textes, maîtres, débats*, ed. I. Rosier-Catach, Turnhout 2011, pp. 605–646 and pp. 647–662, respectively.

[7] Garlandus Compotista, *Dialectica*, ed. L.M. De Rijk, Assen 1959.

[8] De Rijk's hypotheses about the identity of Gerland (*Introduction. Part I: The Author of the Dialectica. His Life and Works*, in Garlandus Compotista, *Dialectica*, op. cit., p. xlv) are, in short, the following: "It has been shown in the first part of this Introduction that the master Gerland

one of the oldest extant in complete form.[9] Recently, based on new findings and improved knowledge of authors and works of the period, Iwakuma has proposed a complete revision of both the attribution of authorship and the period to which this work in fact belongs.[10]

According to Iwakuma, the authorship of the *Dialectica* can be ascribed to one of two Gerlands, an older or a younger. De Rijk attributed it to the former, who is known as Gerland "the Computist." However, the Elder Gerland has now been identified as Saint Gerland, bishop of Agrigento, who died on 25 February 1100. Iwakuma argues that the Younger Gerland, now also credited with authoring the *Candela*,[11] is the author of the *Dialectica*.

Iwakuma's first argument is based on the fact that the earliest records of the vocalist doctrine appear only in 1080. The *Dialectica* exhibits unmistakable traces of its author being a vocalist (an early nominalist); therefore, the work could not have been written before 1075, and certainly not before 1040 as suggested by De Rijk.[12] Iwakuma further argues that if the text had been written during the last two decades of the 11th century, it does not seem likely that the Elder Gerland would have embraced, at an advanced age, such an innovative vision of a topic so well established in the *logica vetus*.[13] On the contrary, the *Dialectica* must have been composed, according to Iwakuma, not before c. 1100 and not after c. 1130.

There are two fundamental lines of evidence for determining this time limitation. The first is that in Gerland's *Dialectica* one can find traces of the *logica nova* and, in particular, indications of some awareness of Aristotle's *Topics*. The

named in the title of the Fleurian manuscript must be the eleventh century computist Garlandus, who was *magister scholarum* at Besançon at the end of his life (c. 1080 A.D.). I propose to call him Garlandus Compotista (c. 1015–before 1102)." A computist is a person who is skilled in computing, for instance, calculating the dates of the calendar using astrometry and celestial mechanics. As De Rijk (*Introduction…*, op. cit., p. xxii) explains: "'Compotus' was the name given in the Middle Ages to what was considered to be the most important branch of astronomical science. It was closely allied to the science of modern almanack-makers, its object being to calculate the Year of Grace and also dates of moveable feasts of the Church, especially that of Easter, by the motions of the sun and the moon. It also served scientific chronology in general."

9 See L.M. De Rijk, *Introduction…*, op. cit., p. xlix; and E. Stump, *Dialectic in the Eleventh and Twelfth Centuries: Garlandus Compotista*, "History and Philosophy of Logic" 1980, Vol. 1, p. 2.

10 See Y. Iwakuma, *"Vocales," or Early Nominalists*, op. cit., pp. 37–111.

11 See ibid., p. 48. *Candela* is an encyclopedic work on dogmatic theology, liturgy, and canon law (cf. L.M. De Rijk, *Introduction…*, op. cit., p. xxxii).

12 De Rijk, when preparing the critical edition of the manuscript, considered that the work should be dated to after c. 1015 and before 1102.

13 Y. Iwakuma, *"Vocales," or Early Nominalists*, op. cit., p. 48.

first texts that document this awareness are the *Logica "Ingredientibus"* of Peter Abelard (c. 1117/1121), the *Ars disserendi* of Adam of Balsham (written in 1132), as well as other texts from the early 12th century, which demonstrate that some scholars had early access to manuscripts of the *Prior Analytics*. Thus Iwakuma concludes that Gerland's *Dialectica* could not have been written before 1100.[14]

The second line of evidence for this dating, Iwakuma argues, lies in certain terminological parallels between Gerland's *Dialectica* and Abelard's *Logica "Ingredientibus"*. The latter work did not circulate before 1120. The use by Gerland of the term *status* in the *Dialectica* is thus coherent with what is found in the texts of the first decades of the 12th century in which this term appears. This employment of the term, however, as Iwakuma explains, is not completely technical, as in Abelard, but it does clearly suggest that Gerland's *Dialectica* predates Abelard's celebrated work. In addition, similar parallels are found between the *Dialectica* and Abelard's *Editio*.[15] Iwakuma argues that "[i]n Abelard's *Editio super Porphyrium* as well as in Gerland's *Dialectica*, the *numerus* is glossed as *collectio* of accidents peculiar to an individual."[16] Therefore, asserts Iwakuma, "[i]f we can conclude from these facts that Abelard had read Gerland's *Dialectica* by the time he wrote his *Editio* in ca. 1102/1108, *Gerland's work should be dated no later than the first years of the twelfth century*."[17]

If Iwakuma's proposed date for the *Dialectica* is correct, its author cannot be the Elder Gerland, the computist, because St Gerland died in 1100. The author would then be, according to Iwakuma, the Younger Gerland, who appears in documents from 1131 to 1134 as prior of the regular canons of St Paul of Besançon.[18] Iwakuma believes that there is no way to definitively decide the authorship; however, he lists other reasons for concluding in favour of Gerland of Besançon.[19]

14 Ibid., pp. 49–50.

15 The authenticity of this work has been questioned. See our observation in sect. 1.

16 Y. Iwakuma, *"Vocales," or Early Nominalists*, op. cit., p. 53.

17 Ibid. (our emphasis).

18 Ibid., pp. 53–54.

19 The rationale for Iwakuma's attribution (ibid., p. 54) takes into account the following: (i) there is a letter from Roscelin to Abelard, dating from when the former was canon of Besançon, Tours, and Loches. At that time, explains Iwakuma, "Gerland may, then, have been a pupil of Roscelin's in Besançon," where he would have had contact with the vocalist doctrine; and (ii) in the *Gesta Alberonis archiepiscopi Balderico* it is recorded that in 1147 the Archbishop of Trèves invited Gerland together with Thierry de Chartres to join him on a trip to Frankfurt. Iwakuma concludes (ibid.): "It might be suggested that Gerland and Thierry had been friends earlier. Both men belonged to the first generation to whom the *logica nova* was accessible. And it may deserve

3. Gerland's *Dialectica*

Gerland of Besançon (died c. 1148) is an important author for understanding the development of certain typically mediaeval logical theories. His work provides an important record of a series of logical doctrines at an early stage of development. De Rijk prepared the edition of the *Dialectica* of Garlandus Compotista from the two remaining manuscripts, Paris BnF Lat. 6438 and Orleans 260(216). According to De Rijk, "Both manuscripts contain the complete text of a *Dialectica* in six books, which, though mainly based on Boethius' logical works, give a rather independent exposition of the *logica vetus*."[20] In this regard, explains Stump, "[d]ialectic in the eleventh to twelfth centuries, though it derives largely from Boethius's work and is couched mainly in his terms, is very different from the method and theory of dialectic in Boethius."[21] Furthermore, Stump writes, "[a]ll of this should give the impression that Garlandus's treatment of dialectic closely resembles Boethius's, and in many respects it does. The dissimilarities, however, are many and important."[22] Thus, although his treatise includes logical doctrines that are standard for the period, Gerland innovates by paying special attention to hypothetical syllogisms, as well as to the study of consequence, a logical doctrine that, as we shall see, he intertwines with the theory of topics.

Gerland's *Dialectica* reflects and portrays the creative dawn of mediaeval scholastic logic.[23] Book by book, the author develops the doctrines of his theoretical framework, placing them at the core of the *logica vetus*, but with hints of the *logica nova*. In the first book, on non-complex voices (*De vocibus incomplexis*), Gerland discusses the categorematic part of language as it relates to logic, analyzing its simplest elements, the non-compound voices (*voces*), as they relate to the

our attention that Thierry is likely to have been a friend of Abelard's as well, since he sided with him at the Council of Soissons in 1121. Is it mere coincidence that Abelard had knowledge both of the *logica nova* and of Gerland's work, as I have argued above?"

[20] L.M. De Rijk, *Introduction…*, op. cit., p. ix. It is worth noting that the sources of the *Dialectica* not only include Boethius's translations and commentaries on the logical treaties of Aristotle, but also works such as Boethius's *De hypotheticis syllogismis*. In the latter figure elements of Megaric-Stoic logic, which are an important starting point for the mediaeval analysis of the conditional proposition, the notion of implication, and logical consequence (see ibid., pp. xlvi–xlix).

[21] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries…*, op. cit., p. 2.

[22] Ibid., p. 3.

[23] Stump (ibid., p. 2) argues that "it [the *Dialectica*] stands at the beginning of the scholastic tradition, and many of the controversies and doctrines of later scholastic work on dialectic are prefigured in or derived from the philosophical tradition represented by Garlandus."

terms that can appear in a proposition, that is, the five predicables of Porphyry and the ten traditional categories of Aristotle; all of these items are understood in the light of a nominalism that is coherent with Gerland's general attitude towards the purpose and object of logic. In the second book, on complex voices (*De vocibus complexis*), Gerland presents the theory of enunciative propositions and their respective types, as well as the main logical operations that can be carried out with these propositions, such as equipollence, opposition, and conversion of categorical propositions.[24] In the third book, on univocal and multiple propositions (*De propositione una et multiplici*), Gerland analyzes the properties of the constitutive elements of the proposition, presenting a semantic theory of univocal propositions (whose terms are univocal) and multiple propositions (whose terms admit multiple meanings); this book also discusses the definition and classification of modal propositions.[25] In the fourth book, on the differences of topics (*De topicis differentiis*), Gerland presents a classification of species of arguments – syllogism, induction, enthymeme, and example – and a series of topics, grouped with an emphasis on their differences.[26] In the fifth book, Gerland discusses categorical syllogisms (*De sillogismis cathegoricis*), presenting them in three figures, analyzing how the valid modes of the second and third figures can be reduced to the valid modes of the first figure. However, it is in the sixth book of the *Dialectica*, dedicated to the study of hypothetical syllogisms (*De sillogismis hipoteticis*), that Gerland reveals interests that anticipate and record, from a historical point of view, certain logico-theoretical tendencies that became more pronounced, and

[24] Gerland does not offer a clear and categorical definition of equipollence. However, from the use of this notion by the author in passages in the *Dialectica*, equipollence corresponds to the logical equivalence relation between two propositions (in today's sense). Regarding the notion of equipollence, Terence Parsons (*Articulating Medieval Logic*, Oxford 2014, p. 60, n. 6) explains: "The term 'equipollent' is not usually defined, but it seems to just mean the logical equivalence between two propositions. So understood, simple conversion produces equipollent propositions, but I am not aware of anywhere that the term is used to characterize the results of conversions."

[25] Garlandus Compotista, *Dialectica*, op. cit., 76.7–14. Throughout this paper we will refer to mediaeval authors in accord with the pattern "x.y–z," where "x" refers to the page number and "y" and "z" denote the corresponding lines.

[26] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 7: "It is plain that for Garlandus, as for Boethius, *Differentiae* are the more important of the two sorts of Topics. *Differentiae* can be thought of, roughly, as the headings under which maximal propositions can be grouped. Some maximal propositions are generalizations about definition, so *definition* (or *from definition*) is a *Differentia*; other maximal propositions are about opposites, so *opposites* (or *from opposites*) is a *Differentia*; and so on."

no less important, in the following centuries. Among them may be highlighted the author's effort to expound the theory of the hypothetical syllogism. In this text, at a very early stage of scholastic logic, Gerland sought by means of topical analysis to establish the logical criteria for the notion of consequence. Even from the point of view of the space it takes up in the text, this discussion stands out: about a third of the *Dialectica* is dedicated to it (Book VI, 63 pages, on hypothetical syllogisms).[27]

Three aspects of the logical theory of the *Dialectica* are important for our discussion: (i) the way the author approaches certain topical inferences with a view to selecting logically acceptable inferences; (ii) his analysis of the notion of logical consequence; and (iii) the definition of implication that may be deduced from his notion of consequence. In what follows, we will try to show whether these three points can or cannot support an interpretation according to which Gerland subtly anticipated discussions about the general invalidity of the *ex falso sequitur quodlibet* principle. This principle, perhaps unknown (or not ostensibly stated) at the time when Gerland was writing, would later be the object of lively controversy in the 12th century.

Before we proceed, it is important to take note of an important stylistic characteristic of the logical treatises of the early scholastics. In the works of both Gerland and Abelard, as in the case of other 12th-century logicians, there is a peculiar literary style, which is markedly different from that found in the exposition of logical theories in the following centuries. In fact, as Brian P. Copenhaver, Calvin G. Normore, and Terence Parsons explain,

> [t]welfth-century logicians […] were disorderly, noisy, and polemical, recording their fights in a genre of books that have been described as catechisms or manifestos. Usually they carry the name of one of the competing schools: they promote the logical principles characteristic of that school; they set problems to be solved by its principles; and sometimes they try to show why a different

[27] For comparative purposes, note that De Rijk's edition of the *Dialectica* totals 181 pages, distributed as follows: Book I, 39 pages; Book II, 21 pages; Book III, 19 pages; Book IV, 28 pages; Book V, 11 pages; Book VI, 63 pages. This outline illustrates the extent to which the sixth book exceeds the size of the others. As Boh explains in *Epistemic Logic in the Later Middle Ages*, London 1993, p. 6: "Observing that Garland's *Dialectica* contains thirty-eight [sic] pages on topics (*loci*), sixty-three pages on hypothetical syllogisms, and only eleven pages on categorical syllogism helps us to realize how important it was to him to find a viable definition or a description of consequence."

> school cannot solve such problems. But logic in this period was belligerent not just in these catechisms but also in textbooks and lecture courses like Abelard's, which often name their rivals and attack rival doctrines.[28]

This polemical and combative characteristic offers additional difficulties in the hermeneutic reconstruction of these authors' logical theories. Thus, in order to recompose this doctrinal mosaic, it is necessary to examine a series of textual elements that, little by little, can allow us to understand the points we wish to focus on in these works.

Firstly, Gerland's formal approach to logic draws our attention, as it anticipates a perspective that would become increasingly common from then on.[29] Thus, De Rijk explains, "[h]e attributed to logic a merely formal task: it only aims, in his opinion, at distinguishing valid arguments from invalid ones and to state *why* they are valid or not. The discovery of truth is of secondary importance to him."[30] Gerland understands logic as *ars sermocinalis*, that is, as a science aimed at analyzing the structure of language rather than defining the status of elements of logic in the science of reality or mind.[31] Second of all, note that the division of dialectics assumed by Gerland maintains links with the previous tradition. In that sense, explains Stump, "Garlandus opens the chapter by paraphrasing Boethius's very imposing division of logic into evaluation of arguments (*judicium*) and discovery of arguments. It is a distinction that Garlandus not only

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[28] B.P. Copenhaver, C.G. Normore, T. Parsons, *Introduction*, in: Peter of Spain, *Summaries of Logic*, Oxford 2014, p. 13. The scholars conclude: "We find nothing like this in Peter's *Summaries*, where the author rarely seems to address an opponent, and then only quietly and obliquely – with a single marginal exception" (ibid.).

[29] Note E. Stump's, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 13, interesting remark concerning the style Gerland uses when analyzing whether some instances are good candidates for being sound logical inferences: "The oddness of Garlandus's examples is mystifying until we see that he is not interested in this or that particular question or conclusion but in the forms of acceptable inferences. He is not interested in settling issues about man's whiteness; his concern is with all inferences of the type 'If every animal is _____, then man is _____', where the same expression is to fill both blanks. Given Garlandus's concern, it is not unlikely that he deliberately chooses apparently inane examples."

[30] L.M. De Rijk, *Introduction*…, op. cit., p. lii.

[31] In this respect, see E.A. Moody, *Truth and Consequence in Mediaeval Logic*, Amsterdam 1953, pp. 5–6; see also P. Thom, *Robert Kilwardby's Science of Logic: A Thirteenth-Century Intensional Logic*, Leiden–Boston, MA 2019, pp. 14–17. Note De Rijk's observation (*Introduction*…, op. cit., p. liii) that "[i]t is indeed a remarkable fact that we do not find any trace of the controversy between realists and nominalists in the *Dialectica* of Garland. He shows himself a consistent nominalist, which can be understood easily because of his conception of logic as an *ars sermocinalis*."

preserves but also takes seriously; and, like Boethius, he relegates dialectic to discovery."[32]

In absorbing the theory of topics from the previous tradition, especially that of Boethius's commentaries on Aristotle's *Topics* and on Cicero's *Topics*,[33] scholastic logic introduces perceptible changes. Contrary to the ancient tradition, in which topics were seen as instruments for the discovery of arguments, topics in scholastic logic are thought of as instruments for the justification of inferences.[34]

Gerland closely follows the systematization of topics in the manner of Boethius, but with notes of originality. For Gerland, topics, understood as *maxima propositio*,[35] are instruments for assessing the validity of arguments.[36] This is an important theoretical feature in the systematization of the logical theories of the period. An epistemic tone dominates in Gerland's definition of argument. On

[32] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 3. It was not by chance that, during the development of scholastic logic, the chapters related to dialectic in logical treatises were absorbed into a general theory of consequence. Stump (ibid., p. 2) explains that "[u]nder its impetus the study of dialectic developed and changed until dialectic became absorbed into the theories of consequences or conditional inferences important in the fourteenth and fifteenth centuries."

[33] *Boethius's "In Ciceronis Topica"*, trans., notes, and essays on the text E. Stump, Ithaca, NJ–London 2004.

[34] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 1, explains that "[b]oth Aristotle and Boethius think of Topics as instruments for a logic of the discovery of arguments. The scholastic use and understanding of Topics is very different from that of Aristotle or Boethius, though the scholastic tradition of dialectic is by no means uniform."

[35] As argued in L. Gili, P. Podolak, *Hugh Eterianus, Alexander of Aphrodisias and Syllogistic Demonstrations: A Newly Discovered Fragment of Alexander of Aphrodisias' "Commentary on Aristotle's Posterior Analytics"*, "Documenti e Studi sulla Tradizione Filosofica Medievale" 2018, Vol. 29, p. 151, Boethius had already translated ἀξίωμα as *maxima propositio*: "In his translation of Aristotle's *Topics*, Boethius had already translated ἀξίωμα in this way (cf. Arist., *Top.* VIII, 1, 155 b15, in: *Topica*, transl. A.M.T.S. Boethius, ed. L. Minio-Paluello, Desclée de Brouwer, Paris 1969, p. 156). […] Accordingly, we maintain that the best English translation of *maxima propositio* is simply 'axiom' in this context."

[36] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 3, explains Gerland's view: "One sort of Topic, he says, is a maximal proposition, which he defines as Boethius does: a maximal proposition is a self-evidently true proposition, for which no proof can be found and which can serve as the basis of proof for other propositions ([*Dialectica*] 87.4–8). Although he says Topics belong to the part of logic concerned with discovery ([*Dialectica*] 86.14), it is important to notice that he assigns maximal propositions to the part of logic concerned with evaluation ([*Dialectica*] 86.12)"; for instance, as Stump (ibid., pp. 3–4) explains, when Garlandus affirms that "a maximal proposition proves a syllogism ([*Dialectica*] 86.12–13)." In this case, Stump (ibid., p. 14) indicates, "'Categorical syllogisms […]', he says, 'are aided by the Topics *from the whole* and *from the part* and *from an equal*' ([*Dialectica*] 114.18)."

this point, he follows Boethius closely, because, as Stump explains, "[i]n his discussion of the nature of an argument, Garlandus (following Boethius) defines argument as a reason producing belief concerning something that was in doubt ([*Dialectica*] 92.19)."[37] For Gerland, the role of an argument is to produce a correct opinion about something that is in dispute or needs to be known. One of his great innovations is the importance placed on conditional propositions throughout the logical theory of the *Dialectica*.[38]

Thus, although the general theory of topics belongs, according to Gerland, to the discovery of the conclusions of the arguments, this is, claims Stump, a subtle change introduced by the mediaeval authors in the topical tradition inherited from the ancients, so that other logical theories, such as the theory of the syllogism (categorical and hypothetical), are subordinated to them; furthermore, of the two types of topics, Gerland, like Boethius, chooses the *differentiae* as the most important item in topical theory. The *differentiae* are, in a general and simplified manner, titles under which maximal propositions are grouped. In what follows, Gerland exhibits the topical foundation of *modus ponendo ponens* and *modus tollendo tollens* and their maximal propositions:

> No one should be surprised if one topic is called by different names, just as the topic "from the adjacencies," which is also called "from common accidentals" by different [authors], while all topics can be named either "from the antecedent" or "[from the] consequent." From the antecedent it is as follows:
>
> "If she gave birth, she lay with a man;
> but she gave birth,
> therefore, she lay with a man,"
>
> the topic from the antecedent: maximal proposition [axiom], once the antecedent is established, the consequent is established.

[37] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 4. The original runs as follows: "Argumentum est ratio rei dubie faciens fidem" (*Dialectica* 92.19).

[38] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 12: "What Garlandus himself is really interested in is the Topics' usefulness for the analysis of conditional propositions. The Differentia finds or provides a conditional premise, and that conditional premise is provided by the maximal proposition in a categorical argument which has the conditional as conclusion."

> From the consequent it is thus:
>
> "but she didn't lie with a man,
> therefore, she didn't give birth,"
>
> the topic from the consequent: maximal proposition [axiom], once the consequent is annihilated, the antecedent is annihilated.
>
> You should know that all the topics are in the service of hypothetical syllogisms; but only the topics "from the whole," "from the part" and "from an equal" are of service to categorical [syllogisms].[39]

Moreover, Stump suggests: "It seems to me just possible that Garlandus is thinking of this broad function of these ultimate maximal propositions when he says that maximal propositions contain or constitute the sense of an argument."[40] Thus Gerland attributes to the topic from the antecedent and to the topic from the consequent, respectively equivalent to the rules of *modus ponens* and *modus tollens*,[41] the important role of grounding *all* hypothetical inferences.[42]

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[39] *Dialectica* 114.3–18, our translation. The original runs as follows: "Nemini mirum videatur, si idem locus diversis nominibus appelatur, sicuti locus ab adiunctis appellatur etiam a communiter accidentibus secundum diversos, cum omnes loci appellari possint ab antecedenti vel consequenti. Ab antecedenti sic: '*si peperit, cum viro concubuit; atqui peperit igitur cum viro concubuit*', locus ab antecedenti; maxima propositio: posito antecedenti ponitur consequens. A consequenti sic: '*sed non concubuit, non igitur peperit*', maxima propositio: destructo consequenti destruitur antecedens. Sciendum est quod omnes loci serviunt hipoteticis sillogismis; cathegoricis vero tantum serviunt locus a toto et a parte et a pari." The propositions employed by Gerland in his examples of *modus ponens* and *modus tollens* come from Cicero's *De inventione* (I.70–72); see Cicero, *On Invention, The Best Kind of Orator, Topics*, trans. H.M. Hubbell, Loeb Classical Library 386, Cambridge, MA 1949, pp. 118–119.

[40] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 14, n. 31.

[41] It should be remembered that both *modus ponens* and *modus tollens* belong to the Stoic tradition, which inaugurated logical-propositional analysis. There they appear as the first and second indemonstrables, respectively. Such results reached the mediaeval authors, as far as we know, mainly through the works of Boethius.

[42] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 14, explains that Garlandus deepened the link between the theory of the topics and of conditional propositions: "Garlandus goes even further along this line by claiming that all Topics can be subsumed under two most general Topics, the Topic *from the antecedent* and the Topic *from the consequent*. Corresponding to each of these is a maximal proposition; as Garlandus gives these, they are equivalent to the rule for *modus ponendo ponens* and the rule for *modus tollendo tollens*, respectively ([*Dialectica*] 114.3–16). These are, of course, basic axioms for all hypothetical arguments; and the fact that Garlandus lists them as Topics strongly suggests that he thinks all hypothetical argumentation is dependent for its validity on the Topics."

4. Gerland's Concept of Consequence

The notion of consequence is fundamental to logic. The search for a general definition of consequence was as important to Gerland as it was to Abelard and other authors of the 12th century. In fact, as Boh explains, "[a]lthough the idea of consequence must be as old as the idea of logic itself, Garland pays more attention to it than the logic textbooks of the *summulists* of the thirteenth century."[43] We can therefore reconstruct the author's perspective from other items of his exposition because, as the scholar explains, "Garland does not offer a definition of consequence as such, but he does discuss truth and falsity conditions of it in some detail, and he is helpful with providing examples."[44] However, one must keep in mind, as Stump argues, that for Gerland "[t]he complete argument would consist of this conditional as a first premise, the assertion of the conditional's antecedent as a second premise, and the consequent of the conditional as the conclusion."[45] Such a structure fits perfectly with the configuration of the rules of *modus ponens* and *modus tollens* – rules of inference that are undeniably Stoic, as previously noted.

As hypothetical inference and its vehicle – the conditional proposition – are central for Gerland, the conditions of truth and falsehood become crucial in order for us to get to the heart of his notion of consequence. He states in the *Dialectica* that a consequence is true in four ways:

[43] I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 4.

[44] Ibid.

[45] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 9. Stump details Gerland's method: "In practice, what Garlandus draws is a conditional proposition. But since the argument he has in mind is a simple hypothetical syllogism, all he really needs to find is a suitable conditional proposition. The question determines the conclusion (or, more precisely, the categorematic terms of the conclusion) since the conclusion must constitute a positive or a negative answer to the question. And given the conditional proposition and the conclusion, the second premise of the argument will be obvious" (ibid.). This strategy is inherently linked to logical-topical methods because, as Stump (ibid., p. 16) explains, "[f]or Boethius, the main function of the Topics is discovery, and what they aid in discovering are third terms around which categorical arguments can be built. For Garlandus, the important function of the Topics is confirmation; and although they confirm all inferences on his view, he is especially interested in them in so far as they confirm enthymematic inferences in conditional propositions and so help determine the truth or falsity of premises in hypothetical syllogisms."

[Gar-T1] One is composed of two true propositions, as in "If Socrates is a man, he is an animal"; for both of these are true, "Socrates is a man" and "Socrates is an animal."

[Gar-T2] Another is composed of two false propositions, as in "If Socrates is a stone, he is inanimate"; for both (components) are false.

[Gar-T3] Another one is composed of a false antecedent and a true consequent, as in "If Socrates is an ox, he is an animal"; for it is false to say that Socrates is an ox and true that he is an animal.

[Gar-T4] Still another is composed of parts neither of which is true or false, such as you can discern in this example: "If it were a man, it would be an animal"; neither of these is true or false.[46]

A close look at these semantic clauses reveals more than can be concluded at first glance, as, for example, claiming that they only govern a classic notion of consequence or logical implication.[47] The first three clauses exemplify a pattern of logical relationship between general terms in which the minor is assimilated into the major and the inferior into the superior, and in which the species is included in the genus and the part in the whole. As Boh explains, a consequence such as "If Socrates is a man, Socrates is an animal" can be demonstrated to be necessary by assuming a first-order premise of the type "Every man is an animal," or, as we

[46] Garlandus Compotista, *Dialectica*, op. cit., 136.28–137.1. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 5. The original runs as follows: "Consequentia quattuor modis sit vera, alia ex utrisque veris, ut ista: '*si Socrates est homo, est animal*' – vere enim utreque sunt '*Socrates est homo*', '*Socrates est animal*' – alie ex utrisque falsis, ut hec: '*si Socrates est lapis, est inanimatum*' – utreque sunt false –, alia ex falso antecedenti et vero consequenti, ut hic: '*si Socrates est bos, et est animal*' – falsum est enim dicere Socratem esse bovem et verum est esse animal –, alia ex utrisque terminis neque veris neque falsis, quem admodum in ista potest dinosci: '*si esset homo, esset animal*': neuter namque veris est neque falsus."

[47] Like other mediaeval authors, Gerland considers, without distinction, conditional propositions to be arguments and arguments to be conditionals. In this sense, explains Stump (*Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 11, n. 19), "[i]t is worth remembering in this connection, though, that in various places Garlandus refers to the conditional premisses alone as arguments; see, for example, [*Dialectica*] 102.20–28, 105.31–32, 106.4–7 and 106.29–31." However, other logicians of the 12th century, Peter Abelard for instance, made a clear distinction between these notions; see C.J. Martin, *William's Machine*, "The Journal of Philosophy" 1986, Vol. 83, No. 10, p. 569.

have already noted, using a result of a metalogical character, such as the topical rule "Of whatever a species is predicated, of it genus is also predicated."[48] In this case, Boh suggests that, "[c]onsidering that 'Socrates' is simply an instance of general terms, we can think here more of a generalized conditional structure of the form: Given that 'man' involves 'animal', it follows that 'If *x* is a man, then *x* is an animal.'"[49] This interpretation may be added to Stump's conclusion[50] that Gerland employs topical rules, the logical properties of conditional and categorical propositions, and the semantic clauses of his notion of consequence, for justifying results in his logical-dialectic theory. Such a practice was not so uncommon, and it demonstrates how mediaeval authors used this theoretical tool to develop their expositions of, and innovations in, logical theory.

We propose, in Table 1, a formalization of Gerland's four statements in a first-order language, where "T" denotes "true"; "F" denotes "false"; "I" denotes "indeterminate"; "⊂" denotes "is strictly included in" (or "is a proper subset of"); "◊" denotes "it is possible that"; "Ms" denotes "Socrates is a man"; "As" denotes "Socrates is an animal"; "Ss" denotes "Socrates is a stone"; "Ins" denotes "Socrates is inanimate"; "Os" denotes "Socrates is an ox"; "◊Mx" denotes "it is possible that x is a man"; "◊Ax" denotes "it is possible that x is an animal"; "M" denotes the "set of all men"; "A" denotes the "set of all animals"; "S" denotes the "set of all stones"; "In" denotes the "set of all inanimates"; "O" denotes the "set of all oxen"; "M◊" denotes the "set whose elements are possibly humans"; and "A◊" denotes

[48] In this regard, Boh (*Epistemic Logic in the Later Middle Ages*, op. cit., p. 10) states: "Otto Bird, in his attempt to scrutinize the whole mechanism of the topics or *loci*, considers the relation of the species to genus as one of class inclusion, and the above maxim as a thesis of the logic of classes:

$$(A \subset B) \rightarrow (\forall x)\ (x \in A \rightarrow x \in B)$$

The consequence 'If it is a man, it is an animal' is simply an instance of it and the *locus* can now be seen as a seat of arguments and the power of inference (*vis inferentiae*)." Boh points out that Stump, the leading investigator of the topical tradition, approves this understanding despite some reservations.

[49] Ibid., p. 4.

[50] Stump affirms (*Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 9): "Garlandus in several places refers to the conditional propositions, given as examples for the *Differentiae*, as arguments (for example, 'If the whole house is white, the wall is also white'). But he also indicates in various places that he thinks of the questions to be settled by Topical arguments as categorical questions, that is, questions in which a categorical proposition is in doubt. These two considerations strongly suggest that the arguments that Garlandus is thinking of, when he says that arguments are drawn from the *Differentiae*, are simple hypothetical arguments consisting of one conditional premise and one categorical premise; these premises together prove the categorical conclusion by *modus ponendo ponens* or *modus tollendo tollens*."

the "set whose elements are possibly animals." Note that "$<X, Y> \Rightarrow Z$" must be understood as "If the truth value of the antecedent is X and the truth value of the consequent is Y, then the truth value of the conditional is Z."

Table 1. Modes of true consequence in Gerland's *Dialectica*

Clause	**Example/statement**	**Valuation**	**Formalization**	**Basis**
Gar-T1	If Socrates is a man, he is an animal	$<T, T> \Rightarrow T$	$Ms \rightarrow As$	$M \subset A$
Gar-T2	If Socrates is a stone, he is inanimate	$<F, F> \Rightarrow T$	$Ss \rightarrow Ins$	$S \subset In$
Gar-T3	If Socrates is an ox, he is an animal	$<F, T> \Rightarrow T$	$Os \rightarrow As$	$O \subset A$
Gar-T4	If it were a man, it would be an animal	$<I, I> \Rightarrow T$	$\Diamond Mx \rightarrow \Diamond Ax$	$M_{\Diamond} \subset A_{\Diamond}$

We claim that the fundamental characteristic of the notion of consequence in Gerland's *Dialectica* is its connexive feature, that is, his notion of logical consequence requires that the consequent of a true declarative conditional sentence must be related to the antecedent, that is, it requires a containment relation between the antecedent and the consequent. This conclusion is solidly based on other passages of the *Dialectica*. It is anchored, for now, in the clause Gar-T4, according to which, even in the case when neither the antecedent nor the consequent is true or false – we are considering that both are indeterminate – the conditional is true.

Decisive elements for the intrinsic connection of the consequence relation in Gerland come to the fore when he introduces the semantic clauses in which the consequence is false:

[Gar-F1] One is false with both components being true, as in "If Socrates is an animal, he is a man."

[Gar-F2] Another consists of two false components, e.g. "If Socrates is inanimate, he is a stone."

[Gar-F3] Still another one is made of false antecedent and true consequent, as in "If Socrates is a stone, he is a man."

[Gar-F4] Another one is composed of two parts, neither of which is either true or false, e.g. "If Socrates were an animal, he would be a man."

[Gar-F5] And still another one is false which has a true antecedent and a false consequent, as in "If Socrates is a man, he is a stone."[51]

Now let us examine Table 2, where "$\not\subseteq$" denotes "is not included in" or "is not a subset of."

Table 2. Modes of false consequence in Gerland's *Dialectica*

Clause	Example/statement	Valuation	Formalization	Basis
Gar-F1	If Socrates is an animal, he is a man	$<T, T> \Rightarrow F$	$As \rightarrow Ms$	$A \not\subseteq M$
Gar-F2	If Socrates is inanimate, he is a stone	$<F, F> \Rightarrow F$	$Ins \rightarrow Ss$	$Ins \not\subseteq S$
Gar-F3	If Socrates is a stone, he is a man	$<F, T> \Rightarrow F$	$Ss \rightarrow Ms$	$S \not\subseteq M$
Gar-F4	If Socrates were an animal, he would be a man	$<I, I> \Rightarrow F$	$\Diamond As \rightarrow \Diamond Ms$	$A_{\Diamond} \not\subseteq M_{\Diamond}$
Gar-F5	If Socrates is a man, he is a stone	$<T, F> \Rightarrow F$	$Ms \rightarrow Ss$	$M \not\subseteq S$

[51] Garlandus Compotista, *Dialectica*, op. cit., VI, 137.4–10. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 4, modified. The original runs as follows: "Quinque modis fit false consequentia: alia ex utrisque veris falsa est, ut ista: '*si Socrates est animal, et est homo*', alia ex utrisque falsis, ut hec: '*si Socrates est inanimatus, est lapis*', alia ex falso antecedenti and vero consequenti, ut hic: '*si Socrates est lapis, est homo*', alia ex utrisque neque veris neque falsis, ut hic: '*si Socrates esset animal, esset homo*', alia iterum fit falsa ex vero antecedenti et falso consequenti, ut hic patet: '*si Socrates est homo, est lapis.*'"

Altogether, the semantic clauses for the veracity and falsity of a consequence or logical implication clearly indicate that Gerland is not dealing with a classical or material notion. When the semantic clauses above are compared, they seem to suggest an incoherent scenario in terms of the definition of consequence. On the one hand, the clauses Gar-T1–T3 and Gar-F5 seem to coincide with the truth conditions of logical-classical material implication. On the other hand, Gar-T1–T4 contrast with and antagonize clauses Gar-F1–F4, which, when considered in detail, demonstrate that it is not the designation of truth values for the antecedent and consequent in conditional propositions that decides the truth of a consequence. Observe that excepting the case in which the antecedent is true and the consequent is false – when the conditional is false[52] – in all the other cases in which the antecedent is true or false and the consequent is true or false – and in the cases when the antecedent and the consequent are "indeterminate" – the conditional may be true or false. That is, the only case in which the truthfulness of the conditional does not depend on the content of the antecedent and of the consequent is case Gar-F5, in which the antecedent is true and the consequent is false.

For Gerland, what determines the "soundness" of the consequence is whether the content of the consequent is properly contained or comprehended (understood), extensionally or intensionally, in the antecedent, as we have tried to denote in the last column of Tables 1 and 2.[53]

Further evidence that Gerland's notion of consequence or logical implication is connexive occurs, according to Boh, in the context of the discussion of Boethius's *De hypotheticis syllogismis*, where Gerland analyzes other types of consequences when arguing:

[52] Garlandus Compotista, *Dialectica*, op. cit., VI, 137.12–13: "Nulla consequentia facta ex vero antecedenti et falso consequenti potest esse vera."

[53] In this regard, Stump (*Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 8, n. 13) argues: "Technically speaking, the Topically-dependent arguments that Garlandus gives are not enthymemes but conditional propositions. By a true conditional, however, Garlandus means more than just a conditional whose consequent is not false if its antecedent is true; he also requires as a criterion for a true conditional that the consequent can be acceptably inferred from the antecedent. The difference between enthymemes and conditional propositions is not a great one, given this view of conditional propositions."

> A connected (*connexa*) hypothetical proposition in which the conjunction "cum" (= when, if) is placed before the antecedent, sometimes has the same force as the one in which "if" (si) is placed before the antecedent; for example, when I say "If a man exists, an animal exists"; for each of these says: "Because (quia) a man exists, an animal exists." However, sometimes it gets another force, as here, "While (cum) the fire is hot, the heaven is round." For here I do not say that because the fire is hot, the heaven is round; rather, I say that heaven is round at the same time as the fire is hot.[54]

In this excerpt, Gerland introduces a division of the consequence that we find in several other authors, starting with Abelard. Gerland distinguishes (1) the accidental consequence (*per accidens*), that is, one of a temporal nature, delimited at a certain time interval, from (2) the consequence according to nature, that is, a natural one. The latter is subdivided into (i) a natural consequence due to the position of the terms (*per positionem terminorum*), as when a genus follows from the species or a cause produces an effect (for example, "When [*cum*] the sun is above the earth, it is day"), and (ii) a natural consequence by the non-positioning of the terms (*per non-positionem terminorum*), as when a species follows from a genus and an effect from a cause (for example, "If [*si*] it is day, then the sun is above the earth") whose reciprocal ("If the sun is above the earth, then it is day") may, in some cases, be true.[55] According to Boh, *cum* denotes a type of temporal consequence and is weaker, from an intensional point of view, than those constructed with *si* and *quia*. In fact, we could compare it to the consequence that was called *ut nunc* (as of now) in later authors.

Deepening the analysis of the connected proposition and the ways in which it can be reduced to other propositions, Gerland states the equipollence between a connected proposition and a disjunctive proposition in the following manner:

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[54] Garlandus Compotista, *Dialectica*, op. cit., VI, 141.7–13. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 5. The original runs as follows: "Propositio connexa in qua ponitur '*cum*', aliquando eandem vim obtinet cum ea in qua preponitur '*si*', ut cum dico: '*si homo est, animal est*'. Aliquando autem alim vim optinet, ut hic: '*cum ignis calidus est, celum rotundum est*': non enim hic dico *quia* ignis calidus est, celum rotundum esse, sed dico celum esse rotundum *eo tempore quo ignis calidus est*."

[55] As in the examples mentioned above; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 6.

> Simple connected propositions are reduced to disjunctions equipollent to them, the disjunction being formed from negation of the antecedent while the consequent remains as it is.[56]

It is surprising that Gerland formulates, at an early stage of scholastic logic, the reduction of implication in terms of the disjunction of the negation of the antecedent and of the consequent, which would be perfectly valid, in truth-functional terms, in what is nowadays called classical propositional logic (in symbols, $P \rightarrow Q \Leftrightarrow \sim P \vee Q$) and is valid if we consider only his clauses Gar-T1–T3 and Gar-F5. However, even if we consider his disjunction as an exclusive disjunction, such equipollence is not in accordance with Gerland's notion of true consequence, in the sense that the content of the consequent must be properly contained or comprehended in the antecedent, as presented in clauses Gar-T1–T4 and Gar-F1–F5.

Thus, in conformity with the logical methods described earlier, Gerland recognizes as valid the equipollence of the following propositions: "If it is a man, it is an animal," "Either it is not a man or else it is an animal," and "Every man is an animal,"[57] which provides Boh with a basis for concluding that "[r]ecognition of this last kind of equipollence tends to support the interpretation of Gerland's conditionals or consequences as stronger than merely truth-functional."[58] Based on Gerland's analysis, Boh claims that "there is even a reason to think that consequence is for him a proposition which, if true, is necessarily true, and if false, necessarily false. If so, he may be at the very beginning of a prominent line of logicians who define consequence by way of an alethic modal notion of possibility, '$P \rightarrow Q =_{df.} \sim\Diamond(P \,\&\, \sim Q)$.'"[59] Boh concisely presents the problem:

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[56] Garlandus Compotista, *Dialectica*, op. cit., VI, 131.2–4. Translated by Ivan Boh; see I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 6. The original runs as follows: "Simplices propositiones connexe in disiunctas sibi equipollentes resolvuntur prima parte coniuncte propositionis destructa, ultima vero manente integra."

[57] Garlandus Compotista, *Dialectica*, op. cit., VI, 131.32–132.2–3: "Equipollent etiam quedam cathegorice quibusdam hipoteticis propositionibus tam connexis quam disiunctis. Ut hic patet: '*si est homo, est animal*', '*aut non est homo, aut est animal*', '*omnis homo est animal*'; he enim eandem veritatem tenent et per unam alie probantur. Ideoque notandum est quod coniuncte atque disiuncte simplices per universales cathegoricas sibi equipollentes probantur: '*si est homo, est animal*', '*aut non est homo, aut est animal*' verificantur per istam in veritate eis consimilem: '*omnis homo est animal*.'"

[58] I. Boh, *Epistemic Logic in the Later Middle Ages*, op. cit., p. 7.

[59] Ibid., p. 6.

> Thus, if we are to envisage any equivalence or equipollence between connective and disjunctive propositions, then we should make sure that if one side is understood modally, the other side must also be so understood. One should not forget that if $P \rightarrow Q$ is to be taken connectively as $\sim\Diamond(P \,\&\, \sim Q)$, it could not be equivalent to a truth functional disjunction of the denial of the antecedent with the consequent, i.e. to $\sim P \vee Q$. A modal disjunction is required.[60]

This is perfectly possible, Boh concludes, because both Gerland and Abelard "are still under the sway of ancient intensional readings of various 'hypothetical' propositions [so as] to be able to meet this requirement."

It is important to emphasize here that we do not agree with Boh's last claim. His interpretation of Gerland's conditional by using modal disjunction, and in terms of strict implication, does not express the containment relation that must hold between the content of the antecedent and of the consequent. In our final remarks, we will argue for a proto-relevantist-paraconsistentist interpretation of Gerland's consequence relation, which is explicitly stated by Peter Abelard – as previously mentioned, the latter will be discussed in a forthcoming paper.

5. Paraconsistent Logic, Relevance, and Paraconsistency

Now we introduce some basic definitions in order to outline the contemporary paraconsistent logical approach.

A theory whose underlying language has a symbol for negation is *inconsistent* if there is a formula of its language such that the formula and its negation are both theorems of the theory; otherwise, the theory is called *consistent*. A theory is *trivial* if all formulas of its language are theorems.

A logical system is *paraconsistent* if it can be the underlying logic for inconsistent but non-trivial theories, which are called *paraconsistent theories*.

Taking into account the practice of Newton C.A. da Costa, Otávio Bueno and Décio Krause, our use of the terms "consistency" and "inconsistency" is syntactical, according to the original metamathematical terminology of David Hilbert and his school.[61]

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[60] Ibid., pp. 6–7.

[61] N.C.A. da Costa, D. Krause, O. Bueno, *Paraconsistent Logics and Paraconsistency*, in: *Philosophy of Logic*, ed. D. Jacquette, Amsterdam 2006, pp. 821–822.

If the underlying logic of a theory is classical logic, or another standard logic such as intuitionistic logic, inconsistency entails triviality, and conversely.[62]

In paraconsistent logics, the scope of the *principle of (non-)contradiction* is in a certain sense restricted; and, in every paraconsistent logic, from a formula and its negation it is not possible, in general, to deduce any formula of its language. Because of this, in such logics the notions of inconsistency and triviality are, in fact, independent notions.

Thus, the *principle ex falso sequitur quodlibet* – "from falsehood, anything follows" (or *ex impossibili sequitur quodlibet* – "from the impossible, anything follows," or *ex contradictione sequitur quodlibet* – "from contradiction, anything follows," or the *principle of Pseudo-Scotus*, nowadays also known as the *principle of explosion*),[63] the law according to which any statement can be proven from a contradiction, is not valid in general in paraconsistent logics.[64]

Relevance logic or *relevant logic* is a kind of non-classical logic developed as an attempt to avoid the paradoxes of material and strict implications. Relevance logic aims to capture aspects of implication that are ignored by the material implication operator in classical truth-functional logic, specifically the fact that the antecedent seems irrelevant to the consequent, that is, the fact that the conclusion seems to have nothing to do with the premise. Relevance logicians claim that it is necessary to capture the notion of relevance between the antecedent and consequent of a true implication.

The motivations and intuitions underlying relevant logic and paraconsistent logic are distinct, but an interesting property of relevant logic is that the *ex falso* is not valid in general. Hence, relevant logics are paraconsistent logics.[65]

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[62] See I.M.L. D'Ottaviano, *On the Development of Paraconsistent Logic and da Costa's Work*, "The Journal of Non-Classical Logic" 1990, Vol. 7, No. 1–2, pp. 89–152.

[63] Cf. sect. 1, n. 2, above, for further information.

[64] See E.L. Gomes, I.M.L. D'Ottaviano, *Para além das colunas de Hércules*…, op. cit.

[65] See A.R. Anderson, N.D. Belnap Jr, *Entailment: The Logic of Relevance and Necessity*, Vol. 1, Princeton, NJ 1975; R. Routley et al., *Relevant Logics and their Rivals*, Atascadero, CA 1982; J.M. Dunn, G. Restall, *Relevance Logic*, in: *Handbook of Philosophical Logic*, Vol. 6, 2nd ed., eds. F. Guenthner, D. Gabbay, Dordrecht 2002, pp. 1–128.

6. Final Remarks

The notion of logical consequence or implication assumed by Gerland is not truth-functional and requires that the premise must be relevant to the conclusion. Gerland's notion of consequence has a connexive feature, that is, his notion demands a containment relation between the antecedent and the consequent.

Our claim is that Gerland's conception of consequence has attributes of a relevance-logical nature, as it relies on the connection between the content of the antecedent and the consequent, a reliance which could impel him to not assume as valid a type of consequence like the *ex falso*, whether in a "categorical" version (*ex falso sequitur quodlibet*), or in a modal version (*ex impossibili sequitur quodlibet*), or in its particular version (*ex contradictione sequitur quodlibet*). Not even the known corollary of the *ex falso*, "the necessary follows from anything" (*necessarium sequitur ad quodlibet*), that is, a necessary proposition follows from any proposition, is found in his exposition; another variant of this statement, presented by Bocheński in his famous history of logic, "the true follows from anything" (*verum sequitur ad quodlibet*), is also not found in Gerland.[66] In fact, there do not seem to appear such typical statements, or even variations of these, in the *Dialectica*, which is in accord with the fact that the role of the intensional aspect in Gerland's logical analysis goes beyond his professed nominalism (vocalism).

Gerland does not admit, as far as we know, that from the false or from a contradiction there follows any other proposition whatever, as the author himself repeatedly rejects this throughout his exposition, stating that certain inconsistencies produce inconvenience between terms and harbour sophistry.[67] In these cases, in the numerous instances where a way out could be an appeal to some form of the *ex falso*, the author reaffirms the inconvenience of the absurdities that are being deduced and decides the logical dilemmas on the basis of a semantic analysis of the terms, on the logico-grammatical constructions involved, and, of course, on the deductive theory that he has embraced.

Gerland seems to suggest that a stricter notion of consequence is necessary. It is in the context of topical inferences and maximal propositions that we may consider that he did not accept the *ex falso*. Relevance-paraconsistent elements seem to orient Gerland's logical options.

[66] J.M. Bocheński, *A History of Formal Logic*, op. cit., p. 342.
[67] Garlandus Compotista, *Dialectica*, op. cit., 146.13–147.26, for example.

A decisive step towards including Gerland of Besançon in the illustrious line of defenders of a notion of consequence that is, by definition, paraconsistent in a broad sense, is presented in clauses Gar-F1, Gar-F2, Gar-F3 and Gar-F4 above. In fact, we have observed that, according to the sets of clauses Gar-T1–T4 and Gar-F1–F5 taken together, excepting the case in which the antecedent is true and the consequent is false – when the conditional is false – in all the other cases in which the antecedent is true or false and the consequent is true or false – and in the cases when the antecedent and the consequent are indeterminate – the conditional may be true or false.

Hence, according to these clauses, there are circumstances in which a false antecedent does not allow any consequence to be inferred, be it true or false, or in which from a set of false (or contradictory) premises there cannot be concluded – validly – any conclusion whatever, whether true or false. It is a tacit presentation of the statement that it is not the case that anything can be concluded from the false and, like a diamond that needs to be cut, it shines forth an *ex falso non sequitur quodlibet* as an implicit corollary to its notion of consequence.

Last but not least, the theoretical tendency represented by Gerland is important for the later development and discussion of these logical theories because, as Stump concludes,

> [t]hese points of contact between the twelfth-century works and Garlandus's treatise are, of course, not nearly enough to conclude that Garlandus was a direct or indirect source for any of the twelfth-century work – he may or may not have been – but they do show at least that the tradition represented by Garlandus was influential for the authors of these works and so for the study of dialectic in the early scholastic period.[68]

The theoretical tradition represented by Gerland of Besançon is in opposition to the statement of the *ex falso sequitur quodlibet*, thus marking out a paraconsistent approach *lato sensu* throughout the later Middle Ages.

Acknowledgement

We thank our colleague Fábio Maia Bertato for fruitful discussion and suggestions during the preparation of this text. The authors would also like to thank three anonymous referees for their comments and suggestions.

[68] E. Stump, *Dialectic in the Eleventh and Twelfth Centuries*…, op. cit., p. 18.

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Summary

An analysis of positions for and against the principle of *ex falso sequitur quodlibet* is essential to the history of a paraconsistent approach in scholastic logic and in Western thought. In this paper we analyze the role that the *Dialectica* of

Gerland of Besançon played in initiating the discussion about the *ex falso* in the 12th century, and we interpret his position as contrary to the acceptance of the principle. We consider Gerland one of the earliest authors to prepare the path and examine properly the role of the *ex falso sequitur quodlibet* principle, making it central in the philosophical context of the time. We adopt the thesis of Józef Maria Bocheński, according to which the formal aspects of logical theory are essential, decisive, and indispensable to a good historiography of logic.

Key words: theory of topics, logical implication, consequence, *logica vetus*, early scholastic logic, Gerland of Besançon, paraconsistency

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